Cooperative relaying protocols and distributed coding schemes for wireless multiterminal networks

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Cooperative Relaying Protocols and Distributed Coding Schemes for Wireless Multiterminal Networks

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Mots-clés : Communication coopérative, codage distribué, réseaux sans fil de relais.

Résumé : avec la croissance rapide des appareils et des applications mobiles, les besoins en débit et en connectivité dans les réseaux sans fil augmentent rapidement. Il est prouvé que les communications coopératives peuvent augmenter significativement l'efficacité spectrale et la fiabilité des transmissions entre les noeuds extrémaux. Le concept de coopération dans un réseau sans fil compte parmi les sujets de recherche les plus actifs en télécommunications, le but étant d'identifier les stratégies de coopération qui maximiseraient les gains en efficacité spectrale et en puissance d'émission. Pour coopérer, les noeuds du réseau partagent leurs ressources (énergie, bande de fréquence, etc. ...) pour améliorer mutuellement leurs transmissions et leurs réceptions. Dans les réseaux sans fil avec relais, les relais sont des noeuds dédiés á améliorer la qualité de la communication entre les noeuds sources et destination. Dans la premiére partie de la thése, nous nous concentrons sur un réseau sans fil avec relais spécifique oú l'ensemble de sources (mobiles) veulent communiquer leurs messages á une destination commune (station de base) avec l'aide d'un ensemble de relais (contexte cellulaire, sens montant). Nous étudions, sur les plans théorique et pratique, un schéma coopératif dans lequel les relais, aprés une durée d'écoute fixée a priori, essayent de décoder les messages des sources et commencent á transmettre des signaux utiles pour ceux qui sont décodés correctement. Ces signaux utiles sont le résultat d'un codage canal-réseau conjoint. Une des limitations du systéme coopératif précédent est précisément que le temps d'écoute des relais est figé et ne peut pas être adapté á la qualité fluctuante (aléatoire) des liens instantanés sources-relais. Pour pallier cette difficulté, nous proposons et analysons, dans une seconde partie de la thése, un schéma de coopération plus avancé oú le temps d'écoute de chaque relais peut être dynamique. Dans ces conditions, un relais bénéficiant d'une meilleure qualité de réception des sources peut commencer á coopérer plus tôt que d'autres relais ayant une qualité de réception moindre. Enfin, dans la troisiéme et derniére partie de la thése, nous considérons la présence d'une information de retour limitée (limited feedback) entre la destination et les sources et les relais, et tentons de caractériser l'efficacité spectrale d'un tel systéme.

Title: Cooperative Relaying Protocols and Distributed Coding Schemes for Wireless Multiterminal Networks

Keywords: Cooperative communication, destributed coding, wireless relay netwrks, network coding, joint network-channel coding.

Abstract: With the rapid growth of wireless technologies, devices and mobile applications, the quest of high throughput and omnipresent connectivity in wireless networks increases rapidly as well. It is well known that cooperation increases significantly the spectral efficiency (coding gain) and the reliability (diversity gain) of the transmission between the nodes. The concept of cooperation in wireless relays network is still one of the most active research topics in wireless communication, scientists are still searching for the optimal coopera-

tion strategies that make the possible gains at the maximum. Cooperation results when nodes in a network share their power and/or bandwidth resources to mutually enhance their transmissions and receptions. In wireless relay networks, the relays are special nodes that are used to improve the quality of communication between the source nodes and the destination nodes. In particular, the use of relays guarantees more efficient and reliable networks. In this work, we focus on a special wireless relay network where a set of sources (mobiles) want to communicate their messages to a common destination (base station) with the help of a set of relays At the beginning of this work, we focused on the cooperative scheme where the relay, after a fixed portion of time, tries to understand (decode) the source's messages and forwards helpful signals for the correctly decoded ones. One of the limitations of the previous cooperative scheme is the fixe listening time of the relays, which cannot be adapted to the quality of the instantaneous sources-relays links. To solve this problem we propose a more advanced cooperative scheme where the listening time of each relay can be dynamic and not fixed in advanced. So the relay that has strong links with the sources can start cooperating earlier than the other relays with weak links. Currently, we are investigating other directions of possible improvements, for example, how can we use feedback signals to improve the efficiency of the network.

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Abstract

With the rapid growth of wireless technologies, devices and mobile applications, the quest of high throughput and omnipresent connectivity in wireless networks increases rapidly as well. A well designed cooperation between complex systems' elements increases significantly the throughput and the efficiency of these systems. The concept of cooperation in wireless relays network is still one of the most active research topics in wireless communication, and scientists are still searching for optimal cooperation strategies that make the possible gains (spectral efficiency, reliability, coverage, etc) at the maximum. Cooperation results when nodes in a network share their power and/or bandwidth resources to mutually enhance their transmissions and receptions. Network coding allows the intermediate nodes to share also their computation capabilities and has grabbed a significant research attention since its inception. Moreover it has become an attractive candidate to bring promising performance improvement in wireless relay networks. Substantial research efforts are currently focused on theoretical analysis, implementation and evaluation of network coding from a physical layer perspective.

In this thesis, we investigate cooperative communication strategies for the slow fading half-duplex Multiple-Access Multiple-Relay Channel (MAMRC), defined as follows: (1) Multiple statistically independent sources communicate with a single destination with the help of multiple relays; (2) Each relay is half-duplex; (3) The links between the different nodes are subject to slow fading and additive white Gaussian noise; (4) Some links may interfere. The Multiple-Access (MA) part of the channel model, described in time, is generic. MA schemes differ depending on how channel uses are allocated to the senders (sources and relays).

Signal superposition at the relays and destination, which comes as a natural consequence of the broadcast property, substantially complicates the design and analysis of cooperative protocols. Orthogonality (in either time, frequency, or code space) is the easiest, albeit most inefficient way to cope with interference. Most previous work dealing with network coding for the slow fading half-duplex MAMRC actually assumes Orthogonal Multiple-Access (OMA). This is not the right approach, however, since, from an information-theoretic viewpoint, OMA on (slow) fading channels is known to be strictly suboptimal compared to Non-Orthogonal Multiple Access (NOMA). In order to cope with error-prone source-to-relay links (due to fading and noise), Selective Relaying (SR) is commonly adopted, meaning that the relays forward a function of the correctly decoded sources' packets to the destination. SR, also known as Selective Decode-and-Forward (SDF), has several advantages, especially under NOMA: (1) It prevents error propagation from the relays to the destination; (2) It

reduces the energy consumption at the relays and the level of interference in the network; (3) It provides a better coding gain than non-selective relaying where the relay cooperates if it decodes all the sources. On the other hand, SR requires Cyclic Redundancy Check (CRC) bits to be appended to each source packet, and additional side information at the destination (e.g., by means of dedicated in-band control signals) to specify when a relay cooperates. As far as network and channel coding are concerned, the objective is to achieve the full diversity order of this channel model and to maximize the coding gain. Several approaches have been proposed. Separate Network-Channel Coding (SNCC) treats network and channel coding separately. Channel coding is used to transform the links of MAMRC into equivalent block-erasure channels. At the network layer, network coding is performed (at block or packet level) on the abstracted physical layer. At the receiver side, channel decoding and network decoding are also performed separately. Although it makes it possible to achieve the full diversity, SNCC is not optimal in terms of coding gain and the suboptimality is further increased with Separate Network Channel Decoding (SNCD). In contrast, the rationale behind Joint Network-Channel Coding (JNCC) is to exploit and optimize channel and network redundancy jointly, even though a clear distinction can remain between channel code design on one side, and network code design on the other side, notably to guarantee structural desired properties to the overall code.

In the first part, we assume no channel state information at the transmitters (sources or relays) and no feedback channel between the nodes. We analyze the individual and common outage events of SR/JNCC/JNCD for the slow fading half-duplex MAMRC, the MA part of the channel model, described in time, being generic. The individual and common outage probabilities serve as lower bounds on the Block/packet Error Rate (BLER) for the proposed SR/JNCC schemes. These bounds are tight for finite codewords length (typically a few hundred channel uses). We also examine the behaviour of the outage probabilities in the high-SNR regime to determine the diversity order of the cooperative protocol. We present different approaches to implement SR/JNCC at the relays. In the first approach, the network coding part is based on linear codes over non-binary Galois field (NBNC). We specify a few constraints that the network code must satisfy for the JNCC to achieve the full diversity. For the channel coding part, turbo codes are used to encode the sources' packets, while punctured convolutional codes are used at the relays to generate extra parity bits. Inspired by the earlier work of Jaggi et al., we then come up with a class of simpler very flexible joint network channel binary codes, referred to as Bit-Interleaved XOR (BI-XOR) based JNCC. This code construction is not provably full diversity but close to full diversity with high probability. For both classes of JNCC, we apply the concise and elegant factor graph formalism to the decoding problems at the relays and destination, the JNCD algorithms being described as instances of the sum-product message passing algorithm.

In the second part of the thesis, we propose to combine two DF protocols, namely Dynamic Decode-and-Forward (DDF) and Selective Decode-and-Forward (SDF). In Dynamic Selective Decode-and-Forward (D-SDF), the relays decide when they switch from listening to forwarding, which represent an obvious advantage compared to (Static) SDF (S-SDF) to cope with the random nature of wireless environments, and notably with asymmetric error-prone Source-to-Relay (S-R) links. In D-SDF, the condition which determines the switching can vary during the transmission and become less stringent than having successfully decoded all the sources, as in standard DDF. After some time, the relays can adopt an opportunistic behaviour and cooperate with any subset of successfully decoded sources. With this additional degree of freedom, sources with poor S-R links will not prevent relays from helping other sources experiencing better link conditions. Our contribution is twofold: Outage behaviour of D-SDF on the one hand, and protocol implementation on the other hand. Regarding the first aspect, we characterize the symmetric individual and common MAMRC outage achievable rates in the case where each (relay) sender employs JNCC and each receiver (relay or destination) implements JNCD. As far as protocol implementation is concerned, we design full-diversity JNCC with optimized coding gain, based on families of rate-compatible multiple distributed turbo codes. We also provide a complete description of JNCD at the receivers (destination), based on the sum-product algorithm.

In the third and last part of the thesis, we propose and investigate cooperative Incremental Redundancy Hybrid-ARQ (IR-HARQ) strategies based on Selective Decode and Forward (SDF) relaying for the slow fading Orthogonal Multiple Access Multiple Relay Channel (OMAMRC). In contrast with the system model used in the first two parts, a limited feedback from the destination to the relays and the sources is allowed. The destination uses feedback messages to control the (re)transmission of the different nodes (relays and/or sources) with the aim of improving both the spectral efficiency and the reliability (increasing the possibility of decoding all the packets of the sources). Time slots are used optimally and non of them is wasted. We show by Monte Carlo simulations based on information outage probabilities that even the simplest feedback strategy relying on common ACK/NACK can improve the throughput of the OMAMRC dramatically compared to the no feedback case. Designing and evaluating practical modulation and coding schemes matched to the described feedback cooperative strategies, i.e., with BLER approaching the information outage probabilities, is a natural future research direction.

Publication

Journal Papers

1. A. Mohamad, R. Visoz, A.O. Berthet, *Cooperative Relaying and Coding Strategies for Wireless Multihop Networks,* submitted to, IEEE Trans. Wireless. Commun., Apr. 2016.

2. A. Mohamad, R. Visoz, A.O. Berthet, *A Novel Cooperative Strategy for Multiple-Access Multiple-Relay Channels: Dynamic Selective Decode-and-Forward,* submitted to, IEEE Trans. Wireless. Commun., Apr. 2016.

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Patents

1. A. Mohamad, R. Visoz, A.O. Berthet, *Method for Transmitting a Digital Signal for a MARC System Having a Dynamic Half-Duplex Relay, Corresponding Program Product and Relay Device,* World Patent Application, Publication Number: WO /2015 /092302, Issue Date: 25-06-2015, Filed on 17-12-2013 by Orange.

2. A. Mohamad, R. Visoz, A.O. Berthet, *Method for Transmitting a Digital Signal for a MARC System Having Plurality of Dynamic Half-Duplex Relays, Corresponding Program Product and Relay Device,* World Patent Application, Publication Number: WO /2015 /092303, Issue Date: 25-06-2015, Filed on 17-12-2013 by Orange.

3. A. Mohamad, R. Visoz, A. O. Berthet, *Method for Transmitting a Digital Signal for a MARC Having a Dynamic Selective Decode-and-Forward Full-Duplex Relay, Corresponding Program Product and Relay Device,* France, Patent n : 200428FR01. 2014.

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6. A. Mohamad, R. Visoz, A. O. Berthet, *Selective Decode-and-Forward For MAMRC with Feedback, Corresponding Program Product and Relay Device,* Filed as a European patent application by France Telecom Sep. 2015.

7. A. Mohamad, R. Visoz, A. O. Berthet, *Dynamic Selective Decode-and-Forward For MAMRC with Feedback and Half-Duplex relays, Corresponding Program Product and Relay Device,* Filed as a European patent application by France Telecom Jan. 2016.

8. A. Mohamad, R. Visoz, A. O. Berthet, *Dynamic Selective Decode-and-Forward For MAMRC with Feedback and Full-Duplex relays, Corresponding Program Product and Relay Device,* Filed as a European patent application by France Telecom Jan. 2016.

Contents

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List of Acronyms

AEP	Asymptotic Equipartition Property
AF	Amplify-and-Forward
ARQ	Automatic repeat request
AWGN	Additive White Gaussian Noise
AWGNC	Additive White Gaussian Noise Channel
BC	Broadcast Channel
BCJR algorithm	L.R. **B**ahl and J. **C**ocke and F. **J**elinek and R. **R**aviv algorithm
BEC	Block Erasure Channel
BICM	Bit-Interleaved Coded Modulation
BFNC	Binary Field Network Coding
BI-XOR	Bit-Interleaved XOR
BLER	Block Error Rate
CC	Convolutional Code
CDF	Cumulative Distribution Function
CF	Compress-and-Forward
CoF	Compute-and-Forward
CRC	Cyclic Redundancy Check
DCC	Distributed Channel Coding
DDF	Dynamic Decode-and-Forward
D-SDF	Dynamic Selective Decode-and-Forward
DNF	DeNoise-and-Forward
DF	Decode-and-Forward
FD	Full-Duplex
FG	Factor Graph
GA	Genie-Aided
GFNC	Galois-Field Network Coding
HARQ	Hybrid Automated Repeat Request
HD	Half-Duplex
IBLER	Individual Block Error Rate
IR	Incremental Redundancy
ISI	Inter-Symbol Interference
JNC	Joint Network Channel
JNCC	Joint Network Channel Coding

JNCD	Separate Network Channel Decoding
JDCD	Joint Distributed Channel Decoding
LAPPR	Log A Posterior Probability Ratios
LAPR	Log A Priori probability Ratio
LAR	Link-Adaptive Regenerative
LDPC	Low Density Parity Check
MAC	Multiple Access Channel
MAMRC	Multiple Access Multiple Relay Channel
MAP	Maximum A Posteriori
MARC	Multiple Access Relay Channel
MCS	Modulation Coding Scheme
MDS	Maximum Distance Separable
MIMO	Multiple Input Multiple Output
MSE	Mean Square Error
NOMA	Non-Orthogonal Multiple Access
NOMARC	Non-Orthogonal Multiple Access Relay Channel
OMA	Orthogonal Multiple Access
OMARC	Orthogonal Multiple Access Relay Channel
PD	Partial Decoding
pdf	probability distribution function
PAPR	Peak-to-Average Power Ratio
PNC	Physical Layer Network Coding
PSK	Phase Shift Keying
QAM	Quadratic Amplitude Modulation
RBC	Relay Broadcast Channel
RSC	Recursive Systematic Convolutional
SDF	Selective Decode and Forward
SINR	Signal to Interference Ratio
SISO	Soft-In Soft-Out
SNCC	Separate Network Channel Coding
SNCD	Separate Network Channel Decoding
SNR	Signal-to-Noise Ratio
SOMAMRC	Semi-Orthogonal Multiple Access Multiple Relay Channel
SoDF	Soft Decode-and-Forward
SOMARC	Semi-Orthogonal Multiple Access Relay Channel
SPA	Sum-Product Algorithm
TC	Turbo Code
TWRC	Two-Way Relay Channel
UE	User equipment
WEF	Weight Enumerator Function

Notations

$\mathbf{x}$	A vector
$\Vert\mathbf{x}\Vert$	Euclidean norm of the vector $\mathbf{x}$
$\mathbf{X}$	A matrix
$x_{i;j,\ell}$ or $[\mathbf{X}_i]_{j,\ell}$	The entry (j, ℓ) of the matrix $\mathbf{X}_i$ with i designating a user
$\det(\mathbf{X}), \vert\mathbf{X}\vert$	Determinant of the matrix $\mathbf{X}$
$\mathrm{diag}(\mathbf{X})$	Diagonal operator on the square matrix $\mathbf{X}$ $\mathrm{diag}(p_1, p_2, \cdots, p_n)$ is a diagonal matrix with the diagonal entries equal to $p_1, p_2, \cdots, p_n$
$(\cdot)^\top$	Transpose operator
$(\cdot)^\dagger$	Complex conjugate transpose / Hermitian operator
$(\cdot)^{-1}$	Inverse operator
$\mathbf{I}_n$	The n-square identity matrix
$\mathbf{0}_n$	n-tuples of zeros
$\mathbf{1}_n$	n-tuples of ones
$\mathcal{S}$	Calligraphic upper case letters are used to denote finite countable sets of the form $\mathcal{S} = \{s_1, \ldots, s_{\vert\mathcal{S}\vert}\}$ where $\vert\mathcal{S}\vert$ is the cardinality of the set $\mathcal{S}$
$\{\emptyset\}$	The empty set
$ind(s)$	The index of an element $s \in \mathcal{S}$
$\vert\mathscr{X}\vert$	Cardinality of the set $\mathscr{X}$
$\mathbf{x}_\mathcal{S}$	The vector $[x_{s_1}, \ldots, x_{s_{\vert\mathcal{S}\vert}}]^\top$
$\underline{\mathbf{x}}_\mathcal{S}$	The vector $[\mathbf{x}_{s_1}^\top, \ldots, \mathbf{x}_{s_{\vert\mathcal{S}\vert}}^\top]^\top$
$\mathbf{x} \sim p(\mathbf{x})$	The random vector $\mathbf{x}$ follows the probability distribution function $p(\mathbf{x})$
$\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$	The circularly-symmetric complex Gaussian distribution with covariance matrix $\mathbf{\Sigma}$
$\exp(\cdot)$	Exponential function
$\log(\cdot)$	Logarithmic function
$\mathbb{E}\{\cdot\}$	Operator of expected value
$\mathbb{1}_{\{C\}}$	The indicator function of the condition C i.e., $\mathbb{1}_{\{C\}} = 1$ if C is true and $\mathbb{1}_{\{C\}} = 0$ otherwise
$\doteq$	*exponential equality*, i.e., we write $f(\gamma) \doteq b$ to denote $\lim\limits_{\gamma\to\infty} \frac{-\log f(\gamma)}{\log \gamma} = b$ and $\dot{\leq}$, $\dot{\geq}$ are similarly defined
$\mathbb{R}$	Set of reals
$\mathbb{C}$	Set of complex numbers
$\mathbb{F}_p$	Finite field (Galois filed) with p elements
$\vert x\vert$	Absolute value of a scalar x
$\lceil x \rceil$	Smallest integer greater than or equal to x
$\lfloor x \rfloor$	Greatest integer less than or equal to x
$x \sim \mathrm{Gam}(\alpha, \beta)$	The random variable x follows a Gamma pdf with shape parameter α and rate parameter β

CHAPTER 1

Introduction

1.1 A Brief overview on cooperative communications and wireless relay networks

The main idea behind cooperative communications emerged from the study of relay channels, initially proposed by van der Meulen in [1, 2]. In these works, general three terminal communication problems have been formulated and bounds have been provided for the capacity of relay channels (i.e., the channel that consists of a source transmitting to a destination with the help of a relay). Later in 1979, Cover and El Gammal published further results on relay channels in [3], where significantly improved inner and outer bounds were derived. This is considered the most prominent work on relay channels up till this date since many of the results still could not be superseded. The authors in [3] have presented structurally different random coding schemes and compared their achievable rates with the min-cut max-flow capacity upper bound, which was established in [4]. In the cooperation scheme [3, Theorem 1], the relay decodes the source message and cooperates with the source to help the destination in decoding. This has given rise to Decode-and-Forward (DF) relaying protocol. In the observation scheme [3, Theorem 6], the relay transmits an estimate (or quantized version) of its observation of the source signal to the destination, using ideas from source coding with side information [5, 6]. This scheme has more often been referred to as Compress-and-Forward (CF) strategy [7]. A general theorem was also presented in [3, Theorem 7] that combines cooperation and observation in a single coding scheme in order to maximize the achievable rates. Other relaying strategies such as Partial Decoding (PD) and Amplify-and-Forward (AF) helped to widen the analysis [7–16]. Furthermore, a variety of contributions including new bounds, power control strategies and some results on half-duplex relaying were proposed in [17].

Multiple relay channels, consisting of single-source, single-destination, and more than one relay, were studied in different contributions [7,12–16,18–20], and their achievable rates with DF, PD and CF were presented in [20]. Relaying have also grabbed particular attention in wireless environments. Among the important contributions are those of Laneman and Wornell addressing the performance of important relaying protocols in wireless environments [21–23]. A number of interesting relaying protocols are also proposed and analyzed in [24–26] including repetition coding, and in [27] including space time coded cooperation. Other complementary contributions come in the form of novel information-theoretic results and new insights into information theoretic (random) coding for relays by Kramer et al. [28] and Chong et al. [29].

The extension to multiple-access and broadcast schemes, where multiple sources or multiple receivers are present, has been considered in a variety of contributions [30–44]:
First, the Multiple Access Relay Channel (MARC) was presented in [7, 30, 45], and the corresponding achievable rate regions with AF, DF and CF were derived. Capacity bounds for the MARC with a half-duplex relay and the corresponding achievable rates with AF, DF, and PD were also investigated in [46]. Additionally, a linear relaying protocol called multi-access AF is analyzed in [31] for the MARC, and shown to be optimal at the high multiplexing gain regime.
On the other hand, a cooperative Relay Broadcast Channel (RBC) has first been studied in [33–35]. The authors considered a network with a single source and two receivers, and introduced two channel models, namely, partially cooperative RBC (only one receiver act as a relay for the other) and fully cooperative RBC (both receivers act as relay nodes for each other). They then derived and compared the corresponding achievable rate regions considering DF. The partially cooperative RBC was further studied in [35] for the case of more than one destination. A third RBC model, called dedicated RBC was introduced and studied in [32], where an additional relay node was inserted into the broadcast channel with the sole function of relaying. Rate regions and upper bounds for the cooperative RBC were further developed in [32, 36]. In parallel, AF for the multi-hop Multiple Access Channel (MAC) and Broadcast Channel (BC) (i.e., no direct links between the sources and the receivers) has been studied in [37], where the optimal power allocation on the relays were presented, as well as the AF relay MAC-BC duality.

Finally, the basic idea of the bidirectional or Two-Way Relay Channel (TWRC) was first presented in [38] for noiseless channels. In TWRC, two nodes exchange their information with the help of a relay. Both broadcast transmission and simultaneous multiple access can be considered in this channel model which have been the subject of several research efforts. In [39–41], the authors derived the achievable rates considering full-duplex nodes, broadcast transmission at all nodes and simultaneous multiple-access of all nodes. In [39, 40] the achievable rates were derived for AF, DF, and CF, while in [41], an achievable rate region for Compute-and-Forward (CoF) relaying strategy was derived. In the CoF strategy [47,48], the relay computes (or decodes) a linear combination of the transmitted messages from the nodes-to-relay MAC. Further contributions are based on the premise of half-duplex nodes, for which we distinguish two main categories: (i) two phase bidirectional relay channel with multiple access broadcast protocol in which the two communicating nodes transmit simultaneously to the relay node during the first phase, and the relay broadcasts to both of them during the second phase. In this model there is no direct link between the two communicating nodes; (ii) three phase bidirectional relay channel with broadcast transmission at all nodes and without multiple access. The first channel model has been considered in [49–53] where the authors derived achievable rates considering different relaying schemes (AF, DF, PD, CF, and CoF). Moreover, the broadcast capacity region was derived in [54], where each receiver node knows perfectly the message intended for the other node. Code design and achievable rates for the second model have been considered in [43,44,55]. Several relaying strategies and their corresponding achievable rates were also discussed in [55] for both categories.

Network coding, initially proposed by Ahlswede et al. in [56], for the graphical networks, is a powerful paradigm where intermediate nodes in a graphical network are allowed not only to route but also to perform algebraic operations on the incoming data flows. A graphical network consists of physically separated point-to-point communication links and modeled by a weighted directed acyclic/cyclic graph. It was shown that the max-flow min-cut upper bound on the capacity of single or multi-packets multi-cast graphical networks is achieved by network coding under the condition that all the destinations should decode all the packets. The proof of achievability was done using random coding (which is not linear in general). In [57, 58], the authors show that linear codes are as good as random codes and achieve the same capacity. It is worth mentioning that the capacity of multi-packets multi-cast graphical networks, for the case where each destination does not require to decode all the packets, is not known and neither the cut-set bound nor the linear network coding are optimal in this case [59]. In [60], it was shown that a distributed random linear network coding approach can achieve the same capacity with probability exponentially approaching one with the code length. These remarkable results has motivated further theoretical and practical research to extend network coding to wireless media, where optimally exploiting interference and broadcast properties is one of the main challenges.
The application of network coding to wireless networks has been investigated in a variety of contributions. From a physical layer perspective, wireless network coding can be used in a variety of contexts, in conjunction with channel coding and source coding. In densely deployed multiterminal networks (e.g. sensor networks) where correlation exists between

the sources, the nodes need to combine source coding, channel coding, and network coding. Several ideas and contributions have been proposed in this matter with the aim of introducing some code designs in a unified framework [61–63]. However, in the case of independent incompressible sources, the combination of network coding and channel coding has received particular research attention, and various contributions in the context of MARC [64, 65], TWRC [66–68], user cooperation [69–72] or cross-packet coding for hybrid ARQ systems [73], have been proposed over the last few years. Here, the primary goal of network coding is to provide reliable and spectrally-efficient transmission over the network. From a pure coding perspective, the challenge is to achieve full diversity and to maximize the coding gain. There are essentially two ways to combine network coding and channel coding: Separate Network Channel Coding (SNCC) and Joint Network Channel Coding (JNCC). In SNCC, channel coding is performed locally and separately for each transmission to transform the noisy channels into erasure-based links. On the network layer, network coding is performed for the erasure-based networks which are provided by the lower layers [74]. SNCC requires Separate Network Channel Decoding (SNCD) at the destination, in which channel decoding is first performed at the physical layer and outputs the estimates to the network decoder. However, in JNCC, we exploit the redundancy of the network code to support the channel code, which can finally improve the coding gain of the system. A Joint Network Channel Decoding (JNCD) is then performed at the destination, in which soft information between the network decoder and the channel decoders is exchanged.

The contributions on practical coding designs for wireless communication are divided into two main categories, namely, DF based network coding in which we come back to the message space at all intermediate nodes to construct the network coded message, and estimate-and-forward based network coding in which the network coded message is constructed without returning to the message space, i.e., it would be an arbitrary function of the noisy linear combinations of the codewords transmitted by the sources. In this work, we focus completely on DF based network coding.

Hausl et al. were amongst the first to describe efficient JNCC for MARC based on Low-Density Parity Check (LDPC) codes [64] or turbo codes [65]. Common to this set of contributions are the hypotheses of (i) time-division half-duplex mode for relaying operation; (ii) orthogonality between all the radio links; (iii) joint selective DF strategy, i.e., the relay cooperates if and only if all the decoded messages are error-free. Concerning the first hypothesis, we know from information theory that half-duplex relaying is basically sub-optimal with respect to full-duplex one, but often retained for practical reasons. Physical constraints (severe attenuation over the wireless channel, insufficient electrical isolation between the transmit and receive circuitry, etc.), complexity and cost considerations, most likely explain the moderate interest for the full-duplex relaying schemes. Concerning the second hypothesis, we have already pointed out that wireless media naturally offer broadcast without an additional cost. This intrinsic property comes at the price of signal superposition (i.e. interference) at all intermediate nodes and at the destination. In order to fight back this impediment, orthogonal medium access (in time, frequency or code space) is often assumed in cooperative communications. If orthogonality greatly simplifies the design of JNCC/JNCD and the performance analysis, it also substantially reduces the spectral efficiency of the proposed systems. Indeed, from an information-theoretic point of view, orthogonal multiple-access is, in general, not optimal for the slow fading (quasi-static) channel, although it may be close to optimal at a very low Signal-to-Noise Ratio (SNR). In [64], the authors also assumed error-free source-to-relay links. To justify this hypothesis, we could imagine a restrictive communication scenario where the relay is very close to, and in line of sight with, the two sources. But even in this case, some decoding errors would occur at the relay, since, in practice, constituent codes used on point-to-point links are never perfect. Furthermore, neither of the above code designs guarantee full diversity. More recently, JNCC based on LDPC codes were presented in [75] where the authors also elaborate on orthogonal links and error-free source-to-relay links. Their purpose is to construct JNCC guaranteeing full diversity, which, in essence, leads on an asymptotical analytical reasoning (with respect to the SNR) and the hardening of slow-fading channels into block erasure channels. In this perspective, the achieved diversity of the proposed JNCC does not depend on the quality of the source-to-relay links and, for the sake of simplicity, the authors assumed error free source-to-relay links. However, their proposed JNCC is not generic in terms of coding choice and the number of sources. Moreover, its coding gain decreases enormously for the case of error-prone source-to-relay links, even if its full diversity structure is maintained. The benefit of JNCC in situations where the relay is not able to decode reliably, has also been addressed in [76]. But the authors assume that the signals transmitted from the relay to the destination are analog, and the coding scheme that they employ is oversimplistic compared to [64, 65, 75]. To forward the analog information at the relay in a bandwidth efficient manner, the quantization (or compression) of the log a posteriori ratios of the relay parity bits, has been investigated in [77, 78], which requires the knowledge of the relay-to-destination channel state at the relay. Besides, in all these contributions, the sources and the relay do not interfere. Similar JNCC designs were also proposed in the case of TWRC with time division multiple-access [66, 67]. In [79], a framework for adaptive network coded cooperation was proposed in which the real-time adaptation of network codes to variant link qualities was mainly addressed. This scheme was further investigated in [80] by taking into account the communication link failures. Designing efficient JNCC schemes for MARC based on Turbo principle was investigated in [81–84]. The assumption of perfect

Sources-to-Relay (S-R) links was removed also not-orthogonal and semi-orthogonal access schemes were considered.

Overall, the aforementioned schemes are designed for small wireless networks with specific topologies, which cannot be easily applied to large multi-terminal wireless networks. Recently, a number of contributions study the application of network coding in Multiple Access Multiple Relay Channel (MAMRC) with M sources, L relays and one destination ($M, L \geq 2$), which is a natural extension of a MARC. Among the first contributions are [74, 85] with the common hypotheses of (i) orthogonality between all the radio links; (ii) error-free source-to-relay links; (iii) separate network channel coding and decoding; (iv) binary network coding schemes. Binary network coding, based on the addition modulo 2 (XOR), is not optimal for networks with multiple relays in terms of diversity gain. This issue has initially been addressed in [86] in the case of MAMRC, and in [87] in the case of two-user cooperative network. The authors demonstrated, through outage probability calculations, that the full diversity can only be achieved by using the algebraic or non-binary network codes. They also considered different possible source-to-relay channel situations (outage or not) and they proved the existence of the full diversity achieving network coding schemes in sufficiently high order alphabets. Their work in [87] has then been generalized in [88] for the case of M-user cooperative network, where the authors proposed an equivalent design, but by considering linear block codes over a non binary finite field. They then demonstrated that their proposed scheme is optimal in terms of the Hamming metric and can increase the diversity order without sacrifice in the system rate. However, in all of the above contributions, the sources and the relays do not interfere and the benefit of efficient JNCC has not been explored. Recently, a JNCC/JNCD design based on LDPC code has been proposed in [89] for MAMRC where the authors considered orthogonal links and non-binary channel and network codes. Their proposed scheme was shown to outperform binary JNCC/JNCD in which all network coding operations are binary XOR. However, the latter is already known to be suboptimal when the number of relays exceeds one. The efficiency of their approach compared to a JNCC/JNCD design in which binary channel codes and non binary network codes are employed, has not been investigated.

Different from Galois-field network coding which was the basis of all the above schemes, a complex-filed network coding has been proposed in [90]. The latter is based on the use of linear constellation precoding vectors drawn from complex-field, and Link-Adaptive Regenerative (LAR) relaying scheme in which the detected symbols at the relay are scaled in power according to the SNR of the source-to-relay and the intended relay-to-destination channels. The authors assumed error-prone links and showed that their proposed scheme guarantee the full diversity in the case of MARC and MAMRC. However, the benefit of their proposed complex-field network coding compared to Galois-field network coding in terms of coding gain remains unclear.

1.2 Motivation and scope of the thesis

The main aim of this work is to design and propose effective cooperation schemes for the half-duplex MAMRC, denoted by $(M, L, 1)$-MAMRC. An $(M, L, 1)$-MAMRC consists of M independent users (or sources) which attempt to transmit their packets to a common destination with the help of L independent relays, where $M \geq 2$ and $L \geq 2$ are arbitrary integers. This channel model could be seen as a generalization of the multiple access relay channel, introduced by Kramer and Wijngaarden [45], and the relay channel, introduced by van der Meulen [2]. The MAMRC could exist in many practical wireless communication scenarios, for example: (1) In wireless cellular networks, where the sources are User Equipments (UEs), the destination is a base station, and the relays are UEs or special devices (fixed or moving), i.e., a relay could be installed on a roof of a bus; (2) In deep space satellite networks, where the sources are deep space satellites (at the borders of our solar system / galaxy, i.e., see the NASA voyager 1,2 missions) with weak wireless links to destination, which could be the earth or (other occupied plant), and the relays could be special satellites or plants with strong wireless links to the destination; (3) In wireless sensors networks, where the sources and relays are sensors, and the destination is the reading collecting center; etc.... The graphical $(M, L, 1)$-MAMRC is a special case of multi-packets multi-cast graphical networks with only one destination, hence its capacity is known and achieved by linear network coding. Unfortunately, the capacity of wireless $(M, L, 1)$-MAMRC in general is still not known [3,7]. Signal superposition at the relays and destination, which arises as a consequence of the broadcast nature of wireless environments, complicates cooperative communications substantially. Orthogonality (in either time, frequency) is the easiest, albeit most inefficient way to cope with interference. Most previous work dealing with network coding for $(M, L, 1)$-MAMRC actually assumes Orthogonal Multiple-Access (OMA). This is not the right approach, however, since, from an information-theoretic viewpoint, OMA on (slow) fading channels is known to be strictly suboptimal compared to Non-Orthogonal Multiple Access (NOMA) [91].

In order to cope with the random nature of wireless environments, more specifically to address the issue of error-prone source-to-relay links, Selective Relaying (SR) is adopted, meaning that the relays forward a function of the correctly decoded sources' packets to the destination. SR has several advantages, especially under NOMA: (1) It prevents error propagation from the relays to the destination; (2) It reduces the energy consumption at the relays and prevent harmful interference in the network; (3) Sources with poor source-to-relay links will not prevent relays from helping other sources experiencing better link conditions. On the other hand, SR requires Cyclic Redundancy Check (CRC) bits to be appended to each source packet, and additional side information at the destination (e.g., by means of an in-band control signal) to specify when a relay cooperates. SNCC/SNCD and JNCC/JNCD, both in combination with SR, were analyzed from an information outage perspective in [92] and [93,94], respectively. Various *linear* SNCC/SNCD were proposed for the slow-fading half-duplex orthogonal $(M, L, 1)$-MAMRC (OMAMRC), operating either on the binary field or high-order Galois fields [89, 95, 96]. In particular, [96] proves that a necessary and sufficient condition for SNCC to achieve the full diversity order is to impose on the network code to be Maximum Distance Separable (MDS). [96] also devotes a brief section to non-orthogonal links (referred to as SO-MAMRC-I in the following). Practical JNCC/JNCD schemes, in combination with SR, were investigated for the slow-fading half-duplex $(M, L, 1)$-OMAMRC in [97] and for the slow-fading half-duplex $(2, 1, 1)$-NOMARC in [81] (degenerated case, single relay). In [89], the authors propose a practical JNCC/JNCD scheme for $(M, L, 1)$-OMAMRC by combining non-binary irregular low-density parity-check channel coding and random GFNC. Perfect (S-R) links were assumed and a sub-optimal Soft-In Soft-Out (SISO) network decoder was proposed, namely *selection updating decoder*.

Dynamic Decode-and-Forward (DDF) is an advanced Decode-and-Forward (DF) protocol where the half-duplex relays decide on their own when to switch from listening to forwarding. DDF was initially proposed for the Multiple-Relay Channel (MRC) (i.e., $(1, L, 1)$-MAMRC) in [13,98]. In [13], the Diversity-Multiplexing Tradeoff (DMT) formulation of [99] was used to study the outage behavior of slow-fading MRC in the high-SNR regime. It was shown that DDF is optimal for low multiplexing gains. In [98], the outage events for a fixed transmission rate were formulated and evaluated. The DMT for the half-duplex DDF relay in the MARC (i.e., $(M \geq 2, 1, 1)$-MAMRC) with single-antenna nodes, has been studied in [100–102], where the observation of the optimality of DDF at low multiplexing gains was confirmed. In this set of contributions, the dynamic relay cooperates *as soon as* it correctly decodes *all* the packets of the sources. In order to cope with the random nature of wireless environments, and notably with *asymmetric error-prone* Source-to-Relay (S-R) links, we propose to combine DDF with Selective Decode-and-Forward (SDF), another advanced DF protocol (see e.g., [94,97,103] and the references therein). In D-SDF, the relays decide when they switch from listening to forwarding as in classical DDF, but the condition which determines the switching can vary during the transmission and become less stringent than having successfully decoded *all* the sources. After some time, the relays can adopt an opportunistic behavior and cooperate with any subset of successfully decoded sources. With this additional degree of freedom, sources with poor S-R links will not prevent relays from helping other sources experiencing better link conditions.

At the physical layer, a special transmission scheme, known as Incremental Redundancy Hybrid ARQ (IR-HARQ), which combines the conventional ARQ with error correction, has been in use since the appearance of 3G wireless technology [104]. IR-HARQ schemes adapt their error code redundancy, based on the receiver's feedback, to varying channel conditions, and thus achieve better throughput performance than ordinary ARQ. Research in limited feedback for relay networks has made significant progress in the past few years. In [105], it was shown that a very low levels of feedback can make simple orthogonal decode-and-forward strategies competitive with dynamic non-orthogonal decode-and-forward [13]. IR-HARQ and relay selection for multiple relay channel has been considered in many works (see for example [106–109]). In [107], an IR-HARQ protocol for the relay channel was proposed were a maximum number of HARQ rounds is considered. In case of successful decoding at the relay, both the relay and the source cooperate to transmit the message to the destination. In [109], A simple and distributed relay selection strategy is considered for multirelay HARQ channels. Then, a nonorthogonal cooperative transmission between the source and selected relay is utilized for retransmission of source data toward the destination, if needed, using space-time codes. In [110], a simple cooperative HARQ protocol suitable for orthogonal multiple access channel (i.e., $(2, 1, 1)$-OMAMRC) was presented on the basis of network coding. Current cooperative IR-HARQ protocols seldom involve the multi-source multi-relays cooperative case. In [111], a relay ordering algorithm based on finite field network coding was proposed. The proposed algorithm consider SNCC/SNCD framework and dose not take into consideration the decoding set of the destination at the end of each retransmission round. Also many time slots is lost when no relay is able to decode all the sources. To overcome the previous drawbacks, we propose to combine SDF relaying with IR-HARQ concept to produce efficient cooperative IR-HARQ strategies that use limited feedback channels from the destination to the sources and the relays and limited feed-forward channels from the relays to the destination.

1.3 Thesis contributions and outline

The main contributions of this thesis can be divided into the following chapters:

In **Chapter 3**, the main contributions can be summarized as follow:

- We analyze the common and individual outage events of SDF/JNCC/JNCD for the slow fading half-duplex $(M, L, 1)$-MAMRC, the MA part of the channel model, described in time, being generic. The common and individual outage probabilities serve as lower bounds on the Block/packet Error Rate (BLER) for the proposed SDF/JNCC schemes. These bounds are tight for finite codewords length (typically a few hundred channel uses) [112]. We also examine the behavior of the outage probabilities in the high-SNR regime to determine the diversity order of the cooperative protocol.

- We present different approaches to implement SDF/JNCC at the relays. The first approach is based on linear network coding over non-binary Galois fields. We specify a few constraints that Non-Binary Galois Field Network Codes (NBGFNCs) must satisfy for the JNCC to achieve the full diversity. Of peculiar interest are the linear Binary Galois Field Network Codes (BGFNCs) derived from the *binary image* of NBGFNCs. For the channel coding part, turbo codes are used to encode the sources' packets, while punctured convolutional codes are used at the relays to generate extra parity bits. More specifically, from the destination persective, each source that have been helped by at least one relay can potentially benefits from an *extended codebook*, namely a multiple turbo code [113]. Inspired by the earlier work of Jaggi et al. [114], we then come up with a class of simpler very flexible joint network channel binary codes, referred to as Bit-Interleaved XOR (BI-XOR) based JNCC. This code construction is not provably full diversity but close to full diversity with high probability.
- For both classes of JNCC, we apply the concise and elegant factor graph formalism to the decoding problems at the relays and destination, the JNCD algorithms being described as instances of the sum-product message passing algorithm [115].

This chapter has led to the following publications:

- A. Mohamad, R. Visoz, A.O. Berthet, *Cooperative Relaying and Coding Strategies for Wireless Multihop Networks,* submitted to IEEE Trans. Wireless. Commun., Apr. 2015.
- A. Mohamad, R. Visoz, A. O. Berthet, *Outage Achievable Rate Analysis for the Non Orthogonal Multiple Access Multiple Relay Channel,* Proc. IEEE WCNC'13, Shanghai, China, Jul. 2013.
- A. Mohamad, R. Visoz, A. O. Berthet, *Outage Analysis of Various Cooperative Strategies for the Multiple Access Multiple Relay Channel,* Proc. IEEE PIMRC'13, London, UK, Sep. 2013.
- A. Mohamad, R. Visoz, A. O. Berthet, *Practical Joint Network-Channel Coding Schemes for Orthogonal Multiple-Access Multiple-Relay Channel* , Proc. IEEE GLOBECOM'14, Austin, TX USA, Dec. 2014.

- A. Mohamad, R. Visoz, A. O. Berthet, *Code Design for Multiple-Access Multiple-Relay Wireless Channels with Non-Orthogonal Transmission ,* Proc. IEEE ICC'15, London, UK, JUN. 2015.

In **Chapter 4**, we combine the concept of selective relaying and DDF to propose a new relaying strategy coined D-SDF. In D-SDF, a relay is not restricted to wait the correct decoding of all the sources in order to start cooperating. Our contribution is twofold: Outage behavior of D-SDF on the one hand, and protocol implementation on the other hand. Regarding the first aspect, we characterize the symmetric individual and common $(M, L, 1)$-MAMRC ϵ-outage achievable rates in the case where each (relay) sender employs JNCC and each receiver (relay or destination) implements JNCD. The symmetric individual ϵ-outage achievable rate is defined as the highest transmission rate of each source such that the probability of any source to be in outage is less or equal to ϵ. The symmetric common ϵ-outage achievable rate is defined as the highest transmission rate of each source such that the probability of a common outage event, which is defined as the event of having at least one source in outage, is less or equal to ϵ. As far as protocol implementation is concerned, we design full-diversity JNCC with optimized coding gain, based on families of rate-compatible multiple distributed turbo codes. We also provide a complete description of JNCD at the receivers (destination), based on the sum-product algorithm [115].

This chapter has led to the following publications:

- A. Mohamad, R. Visoz, A.O. Berthet, *A Novel Cooperative Strategy for Multiple-Access Multiple-Relay Channels: Dynamic Selective Decode-and-Forward,* submitted to IEEE Trans. Wireless. Commun., Apr. 2015.
- A. Mohamad, R. Visoz, A. O. Berthet, *Dynamic Selective Decode and Forward in Wireless Relay Networks,* Proc. IEEE ICUMT'15, Brno, Czech Republic, Oct. 2015.
- A. Mohamad, R. Visoz, A. O. Berthet, *Outage Analysis of Dynamic Selective Decode-and-Forward in Slow Fading Wireless Relay Networks,* submitted to Proc. IEEE ICC'16.

and patents filing:

- A. Mohamad, R. Visoz, A. O. Berthet, *Method for Transmitting a Digital Signal for a MARC System Having a Dynamic Half-Duplex Relay, Corresponding Program Product and Relay Device,* World Patent Application, Publication Number: WO /2015 /092302, Issue Date: 25-06-2015, Filed on 17-12-2013 by Orange.

- A. Mohamad, R. Visoz, A. O. Berthet, *Method for Transmitting a Digital Signal for a MARC System Having Plurality of Dynamic Half-Duplex Relays, Corresponding Program Product and Relay Device,* World Patent Application, Publication Number: WO /2015 /092303, Issue Date: 25-06-2015, Filed on 17-12-2013 by Orange.

In **Chapter 5**, we combine SDF relaying with IR-HARQ concept to produce efficient cooperative IR-HARQ strategies for OMAMRC. In the proposed strategies, the destination uses feedback messages to (1) end the current transmission cycle (frame) if it correctly decode all the packets of the sources; (2) order the best node, that help in minimizing the common outage event, to transmit in the coming retransmission round. A frame has a maximum duration after which an outage will be declared if at least one message is not correctly decode at the destination. The relays inform the destination about their successful decoded messages at the end of each retransmission round using limited feed-forward channels. In our design, we pay particular attention to the fact that all the time slots are always used efficiently. A method to take into consideration the effect of using feed-forward and feedback channels on the throughput is proposed and evaluated numerically in an extreme case scenarios.

- A. Mohamad, R. Visoz, A.O. Berthet, *Cooperative Incremental Redundancy Hybrid-ARQ Strategies for Wireless Relay Channels,* submitted to, IEEE Wireless Commun. Letters, Jan, 2016. (Accepted with major revision, March, 2016).

and patents filing:

- A. Mohamad, R. Visoz, A. O. Berthet, *Selective Decode-and-Forward For MAMRC with Feedback, Corresponding Program Product and Relay Device,* Filed as a European patent application by France Telecom 2015.

Finally, in **Chapter 6**, we conclude this work and pave the way for some future works. This chapter has led to the following patents filing:

- A. Mohamad, R. Visoz, A. O. Berthet, *Method for Transmitting a Digital Signal for a MARC Having a Dynamic Selective Decode-and-Forward Full-Duplex Relay, Corresponding Program Product and Relay Device,* France, Patent n: 200428FR01. 2014.
- A. Mohamad, R. Visoz, A. O. Berthet, *Method for Transmitting a Digital Signal for a MARC Having Plurality of Dynamic Selective Decode-and-Forward Full-Duplex Relays, Corresponding Program Product and Relay Device,* France, Patent n : 200465FR01. 2014.

- A. Mohamad, R. Visoz, A. O. Berthet, *Advanced Soft Selective Decode and Forward Relaying, Corresponding Program Product and Relay Device,* France, Patent n: 200571FR01. 2014.

CHAPTER 2

Preliminaries

In this chapter, we briefly introduce some fundamentals and techniques, used in communication and information theory, which we think are important for the readers of this work.

2.1 Marginal function, factor graphs, and the sum-product algorithm

The origins of Factor Graphs (FGs) lie in coding theory, but they offer an attractive notation for a wide variety of signal processing problems. In particular, a large number of practical algorithms for a wide variety of detection and estimation problems can be derived as summary propagation algorithms. The algorithms derived in this way often include the best previously known algorithms as special cases or as obvious approximations. The two main summary propagation algorithms are the sum-product (or belief propagation or probability propagation) algorithm and the max-product (or minsum) algorithm, both of which have a long history. In the context of error-correcting codes, the Sum-Product Algorithm (SPA) was invented by Gallager [116] as a decoding algorithm for LDPC codes; it is still the standard decoding algorithm for such codes. However, the full potential of LDPC codes was not yet realized at that time. Tanner [117] explicitly introduced graphs to describe LDPC codes, generalized them (by replacing the parity checks with more general component codes), and introduced the min-sum algorithm. Both the sum-product and the max-product algorithms have also another root in coding, viz. the BCJR algorithm [118] and the Viterbi algorithm [119], which both operate on a trellis. Before the invention of turbo coding, the Viterbi algorithm used to be the workhorse of many practical coding schemes. The BCJR algorithm, despite its equally fundamental character, was not widely used; it therefore lingered in obscurity and was independently reinvented several times. The full power of iterative decoding was only realized by the breakthrough invention of turbo coding by Berrou et al. [120], which was followed by the rediscovery of LDPC codes. Wiberg et al. [121,122] observed that the decoding of turbo codes and LDPC codes as well as the Viterbi and BCJR algorithms are instances of one single algorithm, which operates by message passing in a generalized Tanner graph. From this perspective, new applications such as, e.g., iterative decoding for channels with memory also became obvious. In this work, we use the FGs style described in [115], because it is: (1) Suitable for hierarchical modeling ("boxes within boxes"); (2) Compatible with standard block diagrams; (3) Simple formulation of the summary-product message update rule.

Let $x_1, \ldots, x_n$, be a collection of variables, in which, for each i, x_i takes on values in some (usually finite) domain (or alphabet) $\mathcal{A}_i$. Let g be $\mathcal{R}$-valued function of these variables, i.e., a function with domain $\mathcal{S} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$ and codomain $\mathcal{R}$. The domain $\mathcal{S}$ of g is called the *configuration space* for the given collection of variables, and each element of $\mathcal{S}$ is a particular *configuration* of the variables. Assuming that summation in $\mathcal{R}$ is well defined, then associated with every function $g(x_1, \ldots, x_n)$ are n *marginal* functions $g_i(x_i)$. For each $a \in \mathcal{A}_i$, the value of $g_i(a)$ is obtained by summing the value of $g(x_1, \ldots, x_n)$ over all configurations of the variables that have $x_i = a$. This type of sum is so central in FGs operation that a nonstandard notation was introduced to handle it: the “not-sum” or *summary*. Instead of indicating the variables being summed over, we indicate those variables *not* being summed over. For example, if h is a function of three variables x_1, x_2, and x_3, then the “summary for $\{x_2\}$” is denoted by

$$\sum_{\sim\{x_2\}} h(x_1, x_2, x_3) = \sum_{x_1 \in \mathcal{A}_1} \sum_{x_3 \in \mathcal{A}_3} h(x_1, x_2, x_3).$$

In this notation we have

$$g_i(x_i) = \sum_{\sim\{x_i\}} g(x_1, \ldots, x_n).$$

Suppose that $g(x_1, \ldots, x_n)$ factors into a product of several *local functions*, each having some subset of $\{x_1, \ldots, x_n\}$ as arguments; i.e., suppose that

$$g(x_1, \ldots, x_n) = \prod_{j \in \mathcal{J}} f_j(\mathcal{X}_j), \tag{2.1}$$

where $\mathcal{J}$ is a discrete index set, $\mathcal{X}_j$ is a subset of $\{x_1, \ldots, x_n\}$, and $f_j(\mathcal{X}_j)$ is a function having the elements of $\mathcal{X}_j$ as arguments.

Definition (1) *An FG is a bipartite graph that expresses the structure of the factorization (2.1). An FG has a* ***variable node****(empty circles) for each variable x_i, a* ***factor node*** *(filled squares) for each local function f_j, and an edge-connecting variable node x_i to factor node f_j if and only if x_i is an argument of f_j.*

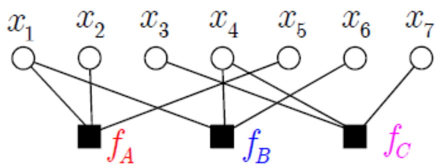


Figure 2.1: An FG for the product in example 1.

An FG is thus a standard bipartite graphical representation of a mathematical relation. In this case, the "is an argument of" relation between variables and local functions.

Example (1) *(A Simple FG) Let $g(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ be a function of seven variables, and suppose that g can be expressed as a product*

$$g(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = f_A(x_1, x_2, x_5) f_B(x_1, x_4, x_6) f_C(x_3, x_4, x_7). \tag{2.2}$$

of three factors, so that $\mathcal{J} = \{A, B, C\}$, $\mathcal{X}_1 = \{x_1, x_2, x_5\}$, $\mathcal{X}_2 = \{x_1, x_4, x_6\}$, and $\mathcal{X}_3 = \{x_3, x_4, x_7\}$. The FG that corresponds to (2.2) *is shown in Fig. 2.1.*

The SPA operates according to the following simple rule:

Definition (2) *(The Sum-Product Update Rule) The message sent from a node v on an edge e is the product of the local function at v (or the unit function if v is a variable node) with all messages received at v on edges other than e, summarized for the variable associated with e.*

Let $\mu_{x \longrightarrow f}(x)$ denote the message sent from node x to node f in the operation of the SPA, let $\mu_{f \longrightarrow x}(x)$ denote the message sent from node f to node x. Also, let $\mathcal{N}(v)$ denote the set of neighbors of a given node v in an FG. Then, the message computations performed by the SPA may be expressed as follows:

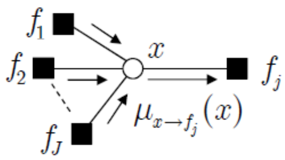
SPA update rule 1: From variable nodes to function nodes (fig. 2.2)

$$\mu_{x \longrightarrow f}(x) = \prod_{h \in \mathcal{N}(x) \setminus \{f\}} \mu_{h \longrightarrow x}(x). \tag{2.3}$$

SPA update rule 2: From function nodes to variable nodes (fig. 2.3)

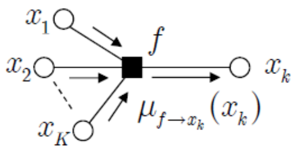
$$\mu_{f \longrightarrow x}(x) = \sum_{\sim \{x\}} \left(f(\mathcal{N}(f)) \prod_{y \in \mathcal{N}(f) \setminus \{x\}} \mu_{y \longrightarrow f}(y) \right). \tag{2.4}$$

Note that variable nodes of degree two perform no computation: a message arriving on one (incoming) edge is simply transferred to the other (outgoing) edge.



$$\boxed{\mu_{x\to f_j}(x) = \prod_{j'\neq j} \mu_{f_{j'}\to x}(x)}$$

Figure 2.2: SPA update rule 1



$$\boxed{\mu_{f\to x_k}(x_k) = \sum_{\sim\{x_k\}} f(x_1, x_2, \ldots, x_k, \ldots x_K) \prod_{k'\neq k} \mu_{x_{k'}\to f}(x_{k'})}$$

Figure 2.3: SPA update rule 2

2.1.1 Modeling systems with FGs

We describe now a way in which FGs may be used to model systems, i.e., collections of interacting variables. In probabilistic modeling of systems, an FG can be used to represent the joint probability mass function of the variables that comprise the system. Factorizations of this function can give important information about statistical dependencies among these variables.

Definition (3) *(Behavior) Let $(x_1, x_2, \ldots, x_n)$ be a collection of variables with configuration space $\mathcal{S} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$. A behavior B is a subset of $\mathcal{S}$ whose elements are called valid configurations.*

Behavioral modeling is natural for codes. If the domain of each variable is some finite alphabet $\mathcal{A}$, so that the configuration space is the n-fold Cartesian product $\mathcal{A}^n$, then a behavior $B \in \mathcal{S}$ is called a *block code* of length n over $\mathcal{A}$, and the valid configurations are called *codewords*.

Definition (4) *(Characteristic function for a behavior) The characteristic (or set membership indicator) function for a behavior B is defined as*

$$\Xi_B(x_1, \ldots, x_n) = \mathbb{1}_{\{(x_1,\ldots,x_n)\in B\}}.$$

Note that Ξ_B could be factored into a product of many characteristic functions, each indicating whether a particular subset of variables is an element of some "local behavior".

Obviously, specifying Ξ_B is equivalent to specifying B.

2.2 Some useful probability distributions, entropy, and mutual information

2.2.1 Gaussian random variable

A real Gaussian random variable (RV), $X \in \mathbb{R}$ is defined as the one having the probability density function (pdf)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-m)^2}{2\sigma^2}}, \tag{2.5}$$

where $m = \mathbb{E}(X)$ is the statistical mean of X, and σ^2 is the variance of X, i.e., $X \sim \mathcal{N}(m, \sigma^2)$. The corresponding cumulative distributed function (cdf) is given by

$$F(x) = 1 - Q(\frac{x-m}{\sigma}), \tag{2.6}$$

where Q is the tail function and defined as

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-t^2/2} dt. \tag{2.7}$$

A complex Gaussian RV $Z = X + jY$, i.e, $Z \in \mathbb{C}$, is one of which $X, Y \in \mathbb{R}$ are real jointly Gaussian RVs.

2.2.1.1 Jointly Gaussian random variables (multivariate Gaussian random vector)

Let $X_1, \ldots, X_n \in \mathbb{R}$ be real valued random variables. They are called *jointly* Gaussian if their joint pdf is given by

$$p(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{K}|}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mathbf{m})^T \mathbf{K}^{-1}(\mathbf{x}-\mathbf{m})\right), \tag{2.8}$$

where $\mathbf{x} = [X_1, \ldots, X_n]^T$, $\mathbf{m} = \mathbb{E}(\mathbf{x})$ is the mean vector, and $\mathbf{K} = \mathbb{E}(\mathbf{x}-\mathbf{m})(\mathbf{x}-\mathbf{m})^T)$ is the covariance matrix. The statement "$X_1, \ldots, X_n$ are jointly Gaussian with mean $\mathbf{m}$ and covariance matrix $\mathbf{K}$" can be compactly written as "$\mathbf{x} \sim \mathcal{N}_n(\mathbf{m}, \mathbf{K})$". Properties of jointly Gaussian random variables include:

- Any subset of jointly Gaussian random variables is also jointly Gaussian;

- Any subset of jointly Gaussian random variables conditioned on any other subset of the original random variables is also jointly Gaussian;
- Jointly Gaussian random variables that are uncorrelated are also independent;
- Linear combinations of jointly Gaussian random variables are also jointly Gaussian. In particular, suppose we produce the vector $\mathbf{y} = [Y_1, \ldots, Y_m]^T$ using the linear transformation $\mathbf{y} = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is an $m \times n$ deterministic matrix. Then,

$$Y \sim \mathcal{N}_m(\mathbf{Am}, \mathbf{AKA}^T).$$

2.2.2 Central Chi-square (Gamma) with k degree of freedom random variable

The pdf of the random variable $X = \sum_{i=1}^{k} {Z_i}^2$, where $Z_i \in \mathbb{R}$ are *zero-mean* statistically independent Gaussian RVs with variance σ^2, i.e., $X \sim Gam(k, 2\sigma^2)$, is

$$p(x) = \frac{x^{\frac{k}{2}-1}}{(2\sigma^2)^{\frac{k}{2}}\Gamma(\frac{k}{2})} e^{\frac{-x}{2\sigma^2}} \quad , x \geq 0, \tag{2.9}$$

where Γ is the gamma function, defined as

$$\Gamma(p) = \int_0^\infty t^{p-1} e^{-t} dt. \tag{2.10}$$

The cdf is given by

$$F(x) = \frac{\gamma(k, \frac{x}{2\sigma^2})}{\Gamma(k)} \quad , x \geq 0, \tag{2.11}$$

where γ is the lower incomplete gamma function defined as

$$\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt. \tag{2.12}$$

When k is even, i.e., $k = 2m$, where m is integer. Then, we obtain

$$F(x) = 1 - e^{\frac{-x}{2\sigma^2}} \sum_{j=0}^{m-1} \frac{1}{j!} \left(\frac{x}{2\sigma^2}\right)^j \quad , x \geq 0. \tag{2.13}$$

The case $k = 2$ yields the exponential distribution RV with pdf

$$p(x) = \frac{1}{2\sigma^2} e^{\frac{-x}{2\sigma^2}} \quad , x \geq 0, \tag{2.14}$$

and cdf

$$F_X(x) = 1 - e^{\frac{-x}{2\sigma^2}} \quad , x \geq 0. \tag{2.15}$$

2.2.3 Rayleigh random variable

The pdf of the random variable $X = \sqrt{Z_1{}^2 + Z_2{}^2}$, where $Z_1, Z_2 \in \mathbb{R}$ are *zero-mean* statistically independent Gaussian random variables with variance σ^2, is

$$p(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} \quad , x \geq 0. \tag{2.16}$$

The cdf is

$$F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}} \quad , x \geq 0. \tag{2.17}$$

Note that the Rayleigh and exponential RV are *equal in distribution* (same cdf).

2.2.4 Circularly-symmetric Gaussian random vectors

Let $\mathbf{z} = [Z_1, \ldots, Z_n]^T \in \mathbb{C}^n$ be complex jointly-Gaussian random vector. By definition, $\mathbf{z}$ is *circularly-symmetric* if $e^{i\phi}\mathbf{z}$ has the same probability distribution as $\mathbf{z}$ for all real ϕ.

Properties of circularly-symmetric Gaussian random vector, i.e. $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{M})$, include:

- $\mathbb{E}(\mathbf{z}) = \mathbf{0}$ ($\mathbf{z}$ must have zero mean).
- $\mathbb{E}(\mathbf{z}\mathbf{z}^T) = \mathbf{0}$ (the pseudo-covariance matrix is zero).
- The pdf

$$p(\mathbf{z}) = \frac{1}{(\pi)^n |\mathbf{M}|} e^{-\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z}}. \tag{2.18}$$

2.2.5 The Entropy

Let X be a discrete random variable with alphabet $\mathcal{X}$ and probability mass function $p(x) = \Pr\{X = x\}, x \in \mathcal{X}$.

Definition (5) *The entropy $H(X)$ of a discrete random variable X is defined by*

$$H(X) = \mathbb{E}_X \left(\log \frac{1}{p(x)} \right) = \sum_{x \in \mathcal{X}} -p(x) \log p(x). \tag{2.19}$$

The entropy $H(X)$ is a measure of the uncertainty of a random variable.

Definition (6) *If $(X, Y) \sim p(x, y)$, the joint entropy $H(X, Y)$, and the conditional entropy $H(X|Y)$ is define as*

$$H(X, Y) = \mathbb{E}_{X,Y} \left(\log \frac{1}{p(x, y)} \right) = - \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(x, y), \tag{2.20}$$

$$H(X|Y) = \mathbb{E}_{X,Y}\left(\log\frac{1}{p(x|y)}\right) = -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y)\log p(x|y), \tag{2.21}$$

respectively.

Some properties of $H(X)$, $H(X,Y)$, and $H(X|Y)$

1. $H(X) \geq 0$;
2. (Chain rule) $H(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} H(X_i|X_{i-1}, \ldots, X_1)$;
3. (Conditioning reduces entropy)(Information can't hurt) $H(X|Y) \leq H(X)$;
4. $H(X) \leq \log|\mathcal{X}|$, with equality if and only if X is distributed uniformly over $\mathcal{X}$;
5. $H(p)$ is concave in p.

The differential entropy $h(X)$ of a continuous random variable X with density $p(x)$ is defined as

$$h(X) = -\int_S p(x)\log p(x)dx, \tag{2.22}$$

where S is the support set of the random variable.

- $h(\mathcal{N}(0,\sigma^2)) = \frac{1}{2}\log(2\pi e\sigma^2)$;
- $h(\mathcal{N}_n(\mathbf{m},\mathbf{K})) = \frac{1}{2}\log((2\pi e)^n|\mathbf{K}|)$.

2.2.6 The mutual information

The mutual information is a measure of the amount of information that one random variable contains about another random variable. It is the reduction in the uncertainty of one random variable due to the knowledge of the other.

Definition (7) *Consider two random variables X and Y with a joint probability mass function $p(x,y)$ and marginal probability mass functions $p(x)$ and $p(y)$. The mutual information $I(X;Y)$ is given by:*

$$I(X;Y) = \mathbb{E}_{X,Y}\left(\log\frac{p(x,y)}{p(x)p(y)}\right) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)}, \tag{2.23}$$

Some properties of $I(X;Y)$:

1. $I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)$;

2. $I(X;Y) \geq 0$, with equality if and only if X and Y are independent;

3. (chain rule) $I(X_1, X_2, \ldots, X_n; Y) = \sum_{i=1}^{n} I(X_i; Y|X_{i-1}, \ldots, X_1)$.

Finally, the mutual information between two continuous random variables with joint density $p(x,y)$ is defined as

$$I(X;Y) = \int p(x,y) \log \frac{p(x,y)}{p(x)p(y)} dxdy. \tag{2.24}$$

2.3 Fading

One of the fundamental problems in wireless communication is multipath, which causes power fluctuations, and multipath delay spread in the received signal. The signal fluctuations cause an increase in the signal power required, relative to steady-signal operation, to achieve the same overall Bit Error Rate (BER) performance. If it occurs in the midregion of the band, frequency-selective fading, can disable proper operation of the modem. Time dispersion of the signal due to multipath puts a limit on the speed at which modulated symbols can be transmitted in the channel. Our analysis in this thesis is based on flat Rayleigh fading, the statistical model most commonly used to describe the behavior of fading on radio channels, a model for which some closed-form solution and simple approximations have been derived. These simple approximations are helpful in gaining an intuitive understanding of the effects of fading on the performance of a modem and how diversity and coding help to improve the performance in fading. In fading channels the received Signal-to-Noise Ratio SNR is a random variable resulting in a bit error rate that is also a random variable. As a result, the performance criterion commonly used for fading channels is either the average error rate over all possible SNR values, or the probability of the error rate exceeding a specified threshold value, and we refer to this as *outage probability*.

2.3.1 Slow fading MAC outage analysis

The slow fading MAC outage analysis is an essential step in the outage analysis of slow fading MAMRC. In this section, unlike [48], we formulate the outage events in a hierarchical and systematic way such that it can help formulating the outage events in the coming chapters of this work. Let us consider the M-users MAC, where a set of statistically independent users $\mathcal{S} = \{s_1, s_2, \ldots, s_M\}$ want to send their messages to a common destination d. Each source s is equipped with one transmit antenna, and the destination is equipped with m_d receive antennas.

Definition (8) *The capacity region of the multiple-access channel is the closure of the set of achievable* $(R_{s_1}, R_{s_2}, \ldots, R_{s_M})$ *rate pairs.*

The received signal at the destination is given by

$$\mathbf{y}_{d,k} = \sum_{s\in\mathcal{S}} \sqrt{\gamma_{s,d}}\mathbf{h}_{s,d}x_{s,k} + \mathbf{n}_{d,k}, \tag{2.25}$$

for $k = 1, 2, \ldots, N$, and $x_{s,k} \in \mathbb{C}$ is the transmitted symbol. The additive noise vectors $\mathbf{n}_{d,k}$ and the channel fading vectors $\mathbf{h}_{s,d}$ are independent identical distributed (i.i.d.) and follow a circularly symmetric complex Gaussian pdf $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$. The channel vectors $\mathbf{h}_{s,d}$ stay constant during the overall transmission and change independently from one transmission to the next (slow fading). $\gamma_{s,d}$ is the average received power at the receiver d from transmitter s. Shadowing and the pathloss can be included in $\gamma_{s,d}$.

The achievable rate region of a $|\mathcal{S}|$-user MAC [123,124] is the complement of the closure of the convex hull of the rate vectors satisfying

$$R_{\mathcal{U}} \leq I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}^c},\ \underline{\mathbf{h}}_{\mathcal{S}d}) \quad \text{for all} \quad \mathcal{U} \subseteq \mathcal{S}, \tag{2.26}$$

where $\mathcal{U}^c = \mathcal{S} \setminus \mathcal{U}$, $R_{\mathcal{U}} = \sum_{s\in\mathcal{U}} R_s$ and given the input distribution $\prod_{s\in\mathcal{U}} p(x_s)$. For the sake of notation simplicity, we remove the channel state from the outage event definitions and mutual information expressions in the following. Let $\mathtt{O}_{d,s}$ denote the individual outage event of source s, and $\mathcal{E}_{d,\mathcal{S}}$ denote the common outage event at the destination d of the $|\mathcal{S}|$-user MAC. Using (2.26) this event could be expressed as

$$\mathcal{E}_{d,\mathcal{S}} = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}^c}) \quad \text{for some} \quad \mathcal{U} \subseteq \mathcal{S}\}, \tag{2.27}$$

equivalently

$$\mathcal{E}_{d,\mathcal{S}} = \bigcup_{\mathcal{U}\subseteq\mathcal{S}} \mathtt{F}_{d,\mathcal{S}}(\mathcal{U}), \tag{2.28}$$

where $\mathtt{F}_{d,\mathcal{S}}(\mathcal{U})$ is defined as the outage event of sources $\mathcal{U}$, the messages of $\mathcal{U}^c = \mathcal{S} \setminus \mathcal{U}$ being perfectly known. This event can be expressed as

$$\mathtt{F}_{d,\mathcal{S}}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}^c})\}. \tag{2.29}$$

When any $\mathtt{F}_{d,\mathcal{S}}(\mathcal{U})$ holds, the destination d can not decode all the messages of $\mathcal{U}$ knowing perfectly the messages of $\mathcal{U}^c$. In this case, a common outage of sources $\mathcal{S}$ is declared. The fact that $\mathcal{E}_{d,\mathcal{S}}$ holds does not mean that the destination can not decode error free the messages of a subset of $\mathcal{S}$. Excluding $\mathcal{S}$ itself and the empty subset, we can define $2^M - 2$ reduced MACs as follows

Definition (9) *A $|\mathcal{I}^c|$-user reduced MAC is a MAC with a subset of sources $\mathcal{I}^c$ of the original MAC, considering the complement of this subset $\mathcal{I} = \mathcal{S} \backslash \mathcal{I}^c$ as interference.*

Definition (10) *An expanded MAC of a $|\mathcal{I}^c|$-user reduced MAC is a MAC that contains at least the $\mathcal{I}^c$ original sources plus one.*

Let $\mathcal{E}_{d,\mathcal{I}^c}$ denote the common outage event of the $|\mathcal{I}^c|$-user reduced MAC. We can express this event by

$$\mathcal{E}_{d,\mathcal{I}^c} = \bigcup_{\mathcal{U} \subseteq \mathcal{I}^c} \mathtt{F}_{d,\mathcal{I}^c}(\mathcal{U}), \tag{2.30}$$

where

$$\mathtt{F}_{d,\mathcal{I}^c}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c})\}, \tag{2.31}$$

defining $\mathcal{U}_c = \mathcal{I}^c \backslash \mathcal{U}$. This equation is very similar to (2.28). However, in $\mathtt{F}_{d,\mathcal{I}^c}(\mathcal{U})$ the set of sources $\mathcal{I}$ are considered as interference, i.e, only the sources belonging to $\mathcal{U}_c$ are supposed to be perfectly known.

Proposition (1) *The source s is in outage iff the $|\mathcal{S}|$-user MAC and all the reduced MAC containing s are in outage.*

$$\mathtt{O}_{d,s} = \bigcap_{\mathcal{I} \subset \mathcal{S} : s \in \mathcal{I}^c} \mathcal{E}_{d,\mathcal{I}^c}. \tag{2.32}$$

proof : The sufficient part: if all the reduced MAC containing s are in outage then the message of s can not be decoded (error free) by any possible mean, thus, the source s is in outage. The necessary part: if the source s is in outage and one reduced MAC including this source is not in outage, it means that the destination can jointly decode the sources of this reduced MAC, as a result, the destination can decode the message of user s which contradicts the statement that s is in outage. ■

In some cases, i.e., at the relays, we are interested in finding $\mathcal{S}_d$, the maximum set of sources that the destination can decode.

Proposition (2) *The sufficient and necessary condition for a set of sources to be $\mathcal{S}_d$ is (i) the $|\mathcal{S}_d|$-user reduced MAC is not in outage and (ii) all the expanded MAC of this $|\mathcal{S}_d|$-user reduced MAC are in outage.*

proof : First, the fact that the $|\mathcal{S}_d|$-user reduced MAC is not in outage is a sufficient and necessary condition for the set $\mathcal{S}_d$ to be jointly decoded error free. In order to ensure that it is the maximum set, any other sub-set with higher cardinality cannot be decoded jointly error free. This is guaranteed by the second condition of the proposition. ■

Finally, the source s individual and common outage probability can be expressed as

$$P^s_{out,d}(\mathbf{R},\boldsymbol{\gamma}) = \int_{\mathbf{h}} \mathbf{1}_{\{\mathtt{O}_{\mathbf{d},\mathbf{s}}(\mathbf{h})\}}\mathbf{p}(\mathbf{h})\mathbf{d}(\mathbf{h}), \tag{2.33}$$

and

$$P^c_{out,d}(\mathbf{R},\boldsymbol{\gamma}) = \int_{\mathbf{h}} \mathbf{1}_{\{\mathtt{E}_{\mathbf{d},\mathcal{S}}(\mathbf{h})\}}\mathbf{p}(\mathbf{h})\mathbf{d}(\mathbf{h}), \tag{2.34}$$

respectively, where $\mathbf{h} = \underline{\mathbf{h}}_{\mathcal{S},d}$, $\boldsymbol{\gamma} = [\gamma_{s_1,d},\dots,\gamma_{s_M,d}]^\top$, **and** $\mathbf{R} = [R_{s_1},\dots,R_{s_M}]^\top$.

Example (2) *Fig. 2.4 shows an example of a capacity region of 3-users MAC for a given channel realization, and a rate vector* $\mathbf{R} = (R_{s_1}, R_{s_2}, R_{s_3})$ *witch lies outside this capacity region. In this case, the destination node can not decode all the sources correctly and the common outage event* $\mathtt{E}_d$ *will be declared. By checking the reduced MACs of cardinality two, where the third source is interference, we can see that none of them can be decoded correctly. Only the reduced MAC of cardinality one witch contains* s_2 *can be decoded error-free. Hence, in this case, the individual outage event of sources* s_1 *and* s_3 *will be declared while the individual outage event of* s_2*, i.e.,* $\mathtt{O}_{d,s_2}$*, will not.*

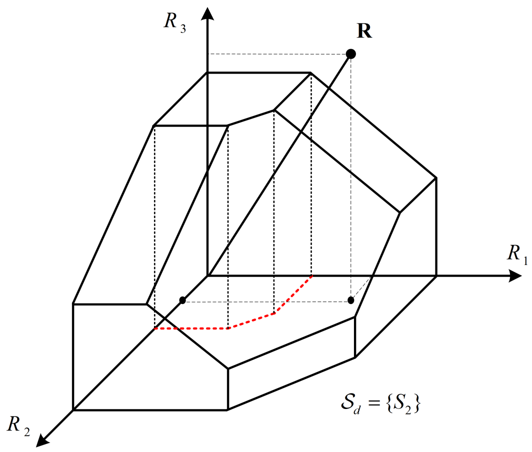


Figure 2.4: An example of achievable rate region of 3-users MAC

Identical rate ($R_s = R, \forall s$): In this case, the common and the individual outage events can be written as

$$\mathtt{E}_d = \{R > \min_{\mathcal{U}\subseteq\mathcal{S}} \frac{I(\mathbf{x}_{\mathcal{U}};\mathbf{y}_d|\mathbf{x}_{\mathcal{U}^c})}{|\mathcal{U}|}\}, \tag{2.35}$$

and

$$\mathtt{O}_{d,s} = \{R > \max_{\mathcal{I}\subset\mathcal{S}} \min_{\mathcal{U}\subseteq\mathcal{I}^c:s\in\mathcal{U}} \frac{I(\mathbf{x}_{\mathcal{U}};\mathbf{y}_d|\mathbf{x}_{\mathcal{U}_c})}{|\mathcal{U}|}\}. \tag{2.36}$$

2.3.2 Point-to-point slow fading channel

Clearly, the point-to-point channel is a special case of the multiple access channel where $M = 1$. Conditioned on the channel realization $\mathbf{h}_{a,b}$ of the direct link between a transmitting node a and a receiving node b, the instantaneous channel is an AWGNC with instantaneous mutual information

$$C_{a,b} = I(x_a; \mathbf{y}_b, \mathbf{h}_{a,b}). \tag{2.37}$$

An outage event occurs when the transmission rate R_a of node a is higher than $C_{a,b}$. Thus, The probability of the outage of the direct link between a and b is defined as

$$P_{out}^{a,b} = \Pr\{R_a > C_{a,b}\} \tag{2.38}$$

The diversity order in a Rayleigh (or Rician) fading environment can be defined as the asymptotic slope, at high SNR, of the error probability curve on a log-log scale [99] and is defined as

$$D_{a,b} = \lim_{\gamma_{a,b}\to\infty} \frac{-\log P_{out}^{a,b}}{\log(\gamma_{a,b})}. \tag{2.39}$$

For Gaussian (i.i.d) input and infinite block length, we have

$$C_{a,b} = \log(1 + \gamma_{a,b}||\mathbf{h}_{a,b}||^2), \tag{2.40}$$

and

$$P_{out}^{a,b} \overset{(a)}{=} 1 - e^{\frac{\epsilon_a}{\gamma_{a,b}}} \sum_{j=0}^{m_b-1} \frac{\left(\frac{\epsilon_a}{\gamma_{a,b}}\right)^j}{j!}, \tag{2.41}$$

where $\epsilon_a = 2^{R_a} - 1$ is constant with respect to $\gamma_{a,b}$, (a) follows from the fact that $||\mathbf{h}_{a,b}||^2 \sim Gam(2m_b, 1)$. From (2.41), we have $D_{a,b} = m_b$.

For uniform distributed discrete inputs, where x_a is chosen from a discrete constellations $\mathcal{X}_a \in \mathbb{C}$ of order 2^{q_a}, we have

$$C_{a,b} = q_a - \sum_{x\in\mathcal{X}_a} \frac{1}{2^{q_a}} \mathbb{E}_{\mathbf{n}_{a,b}} \left(\log_2 \sum_{\tilde{x}\in\mathcal{X}_a} e^{-||\mathbf{n}_{a,b}+\mathbf{h}_{a,b}(x-\tilde{x})||^2+||\mathbf{n}_{a,b}||^2} \right) \tag{2.42}$$

2.4 Graphical networks

Graphical networks are modeled by a *weighted directed acyclic* graph. This network model represents, for example, a wired network or a wireless mesh network operated in time or frequency division, where the nodes may be servers, handsets, sensors, base stations, or routers. In this section, we summarize the limits on communication of independent messages over networks modeled by a weighted directed acyclic graph. In graphical networks, some nodes can act as both senders and receivers and hence communication can be performed over multiple rounds. The edges in the graph represent point-to-point communication links that use channel coding to achieve close to error-free communication at rates below their respective capacities. We assume that each node wishes to communicate a message to other nodes over this graphical network. The nodes can also act as relays to help other nodes communicate their messages. What is the capacity region of this network?

Although communication over such networks is not hampered by noise or interference, the conditions on optimal information flow are not known in general. The difficulty arises in determining the optimal relaying strategies when several messages are to be sent to different destination nodes. We show the cases where the capacity is known.

2.4.1 Capacity of graphical unicast network

Consider a Graphical network modeled by a weighted directed acyclic graph $G = (\mathcal{N}, \mathcal{E}, \mathcal{C})$, where $\mathcal{N} = [1 : N]$ is the set of nodes, $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of edges, and $\mathcal{E} = \{C_{ij} : (i, j) \in \mathcal{E}\}$ is the set of edges wight. In unicast Network, a source node $s \in \mathcal{N}$ wishes to communicate a message $M \in [1 : 2^{nR}]$ to a destination node $d \in \mathcal{N}\backslash\{s\}$. Each node $k \in \mathcal{N}\backslash\{s, d\}$ can also act as a relay to help the source node communicate its message to the destination nodes. Note that in addition to being noiseless, this network model does not allow for broadcasting or interference. However, we do not assume any constraints on the functions performed by the nodes; hence general relaying operations are allowed.

Definition (11) (Cutset bound). *We define a cut $(\mathcal{S}, \mathcal{S}^c)$ as a partition of the set of nodes $\mathcal{N}$. The maximum possible flow of the cut is defined as*

$$C(\mathcal{S}) = \sum_{\substack{(k,l)\in\mathcal{E} \\ k\in\mathcal{S}, l\in\mathcal{S}^c}} C_{kl}, \tag{2.43}$$

Theorem (1) (Max-Flow Min-Cut Theorem). *The capacity of the graphical unicast network $G = (\mathcal{N}, \mathcal{E}, \mathcal{C})$ with source node s and destination node d is*

$$C = \min_{\substack{\mathcal{S}\in\mathcal{N} \\ s\in\mathcal{S}, d\in\mathcal{S}^c}} C(\mathcal{S}), \tag{2.44}$$

Remark: The capacity of a unicast graphical network is achieved error-free using routing. Hence, information in such a network can be treated as water flowing in pipes or a commodity transported over a network of roads.

2.4.2 Capacity of graphical multicast network

The cutset bound turns out to be achievable also when the network has more than one destination. Unlike the unicast case, however, it is not always achievable using only routing. Network coding ensures the achievablity. In multicast graphical network, a source node $s \in \mathcal{N}$ wishes to communicate a message to a set of destination nodes $\mathcal{D} \in \mathcal{N}\backslash\{s\}$. Each node $k \in \mathcal{N}\backslash\mathcal{D}\backslash\{s\}$ can also act as a relay to help the source node communicate its message to the destination nodes.

Theorem (2) *(Network Coding Theorem) The capacity of the graphical multicast network $G = (\mathcal{N}, \mathcal{E}, \mathcal{C})$ with source node s and destination set $\mathcal{D}$ is*

$$C = \min_{j \in \mathcal{D}} \min_{\substack{\mathcal{S} \in \mathcal{N} \\ s \in \mathcal{S}, \mathcal{D} \in \mathcal{S}^c}} C(\mathcal{S}) \tag{2.45}$$

2.4.3 Graphical multimessage multicast network

We now consider the more general problem of communicating multiple independent messages over a network. Let $[1:k]$, for some $k \leq N-1$, be the set of source nodes and assume that the set of destination nodes $\mathcal{D} \in [k+1:N-1]$ is the same for every source. Hence in this general multimessage multicast setting, every destination node in $\mathcal{D}$ is to recover all the messages. The cutset bound is again tight for this class of networks and is achieved via linear network coding.

Theorem (3) *The capacity region of the graphical multimessage multicast network $G = (\mathcal{N}, \mathcal{E}, \mathcal{C})$ with source nodes $[1:k]$ and destination nodes $\mathcal{D}$ is the set of rate tuples $(R_1, \ldots, R_k)$ such that*

$$\sum_{j \in \mathcal{S}} R_j \leq C(\mathcal{S}), \tag{2.46}$$

for all $\mathcal{S} \subset \mathcal{N}$ with $[1:k] \cap \mathcal{S} \neq \emptyset$ and $\mathcal{D} \cap \mathcal{S}^c \neq \emptyset$.

It can be easily checked that for k = 1, this theorem reduces to the network coding theorem.

2.5 General multi-terminal networks

Consider a general multi-terminal network consists of N nodes, each node i has an associated transmitted variable X_i and a received variable Y_i. We denote the set of nodess by $\mathcal{N} = \{1, 2, \ldots, N\}$. The node i sends information at rate R_{ij} to node j. We assume that all the messages W_{ij} being sent from node i to node j are independent and uniformly distributed over their respective ranges $\{1, 2, \ldots, 2^{nR_{ij}}\}$. The channel is represented by the channel transition function $p(y_{\mathcal{N}}|x_{\mathcal{N}}) = p(y_1, \ldots, y_N|x_1, \ldots, x_N)$, which is the conditional probability mass function of the outputs given the inputs. This probability transition function captures the effects of the noise and the interference in the network. The channel is assumed to be memoryless (i.e., the outputs at any time instant depend only the current inputs and are conditionally independent of the past inputs). The upper bound on the rate of flow of information from nodes in $\mathcal{S}$ to nodes in $\mathcal{S}^c$ is given by

Theorem (4) *If the information rates $\{R_{ij}\}$ are achievable, there exists some joint probability distribution $p(x_{\mathcal{N}})$ such that*

$$\sum_{i\in\mathcal{S}, j\in\mathcal{S}^c} R_{ij} \leq I(X_{\mathcal{S}}; Y_{\mathcal{S}^c}|X_{\mathcal{S}^c}) \quad \textit{for all} \quad \mathcal{S} \subset \mathcal{N}. \tag{2.47}$$

Proof. The proof follows the same lines as the proof of the converse for the multiple access channel, see [123, Theorem 15.10.1]. □

The theorem has a simple max-flow min-cut interpretation. The rate of flow of information across any boundary is less than the mutual information between the inputs on one side of the boundary and the outputs on the other side, conditioned on the inputs on the other side. The problem of information flow in networks would be solved if the bounds of the theorem were achievable. But unfortunately, these bounds are not achievable even for some simple channels. We now apply these bounds to channels of interest.

2.5.1 Relay network channel capacity upper bound

In relay network channel (fig. 2.5), a source s wants to send its message to a destination d with the help of L relays. The set of relays is denoted by $\mathcal{R} = \{r_1, \ldots, r_L\}$. The source s, and each relay node $r \in \mathcal{R}$ have the associated transmitted variable denoted by X_s, and X_r, respectively. The destination and each relay node has the associated received variable denoted by Y_d, Y_r, respectively.

Theorem (5) *The relay network channel capacity satisfies*

$$C \leq \max_{p(x_s, x_{\mathcal{R}})} \min_{\mathcal{T}\subseteq\mathcal{R}} I(X_s, X_{\mathcal{T}}; Y_{\mathcal{T}^c}, Y_d|X_{\mathcal{T}^c}). \tag{2.48}$$

Proof. The relay channel is a special case of the multi terminal network with $N = L + 2$, $X_s = X_1$, $Y_s = 0$, $X_{r_i} = X_{i+1}$, $Y_{r_i} = Y_{i+1}$, $X_d = 0$, and $Y_d = Y_N$. Hence, this theorem is a direct application of theorem 4. □

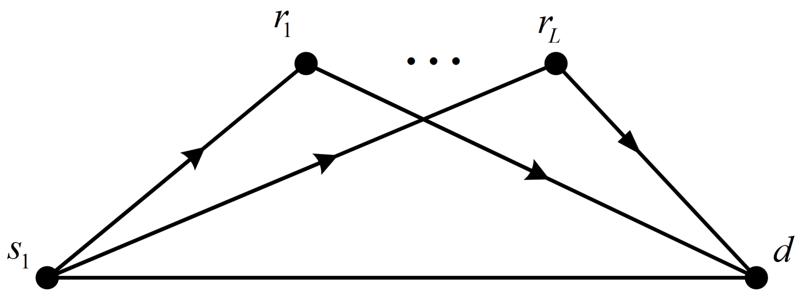


Figure 2.5: The relay network channel

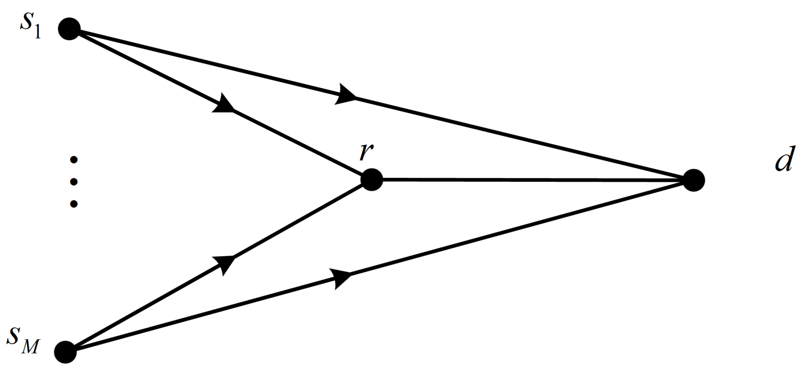


Figure 2.6: The Multiple Access Relay Channel (MARC)

2.5.2 Relay channel capacity upper bound

For example, for $L = 1$ the bound (2.48) becomes

$$C \leq \max_{p(x_s, x_r)} \min\{I(X_s, X_r; Y_d), I(X_s; Y_r, Y_d | X_r)\}. \tag{2.49}$$

This upper bound is the capacity of :

- Physically degraded relay channel, i.e., $p(y_d, y_r | x_s, x_r) = p(y_d | y_r, x_r) p(y_r | x_s, x_r)$, achieved by Decode-and-Forward (DF).

- Orthogonal sender components relay channel. i.e., $p(y_d, y_r | x_s, x_r) = p(y_d | x'_s, x_r) p(y_r | x''_s, x_r)$, achieved by Partial Decode-and-Forward see [125].

- Relay channel with feedback.

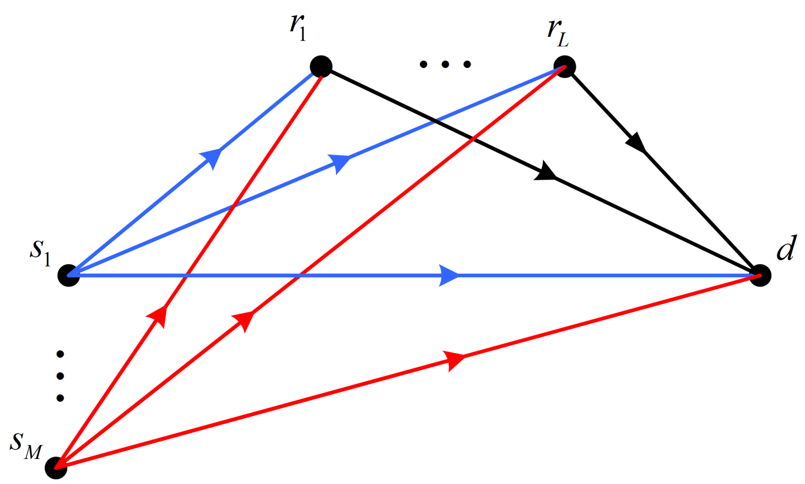


Figure 2.7: The Multiple Access Multiple Relay Channel (MAMRC)

2.5.3 Multiple access relay channel capacity upper bound

In Multiple Access Relay channel (MARC) (fig. 2.6), a set of sources $\mathcal{S} = \{s_1, s_2, \ldots, s_M\}$ want to send their messages to a common destination d with a help of one relay r.

Theorem (6) *If the information rates $\{R_s\}, \forall s \in \mathcal{S}$ are achievable, there exists some joint probability distribution $p(x_\mathcal{S}, x_r)$ such that*

$$\sum_{s \in \mathcal{U}} R_s \leq \min\{I(X_\mathcal{U}, X_r; Y_d | X_{\mathcal{U}^c}), I(X_\mathcal{U}; Y_r, Y_d | X_{\mathcal{U}^c}, X_r)\} \quad \textit{for all} \quad \mathcal{U} \subseteq \mathcal{S}. \tag{2.50}$$

Proof. The MARC is a special case of the multiterminal network with $N = M+2$, $X_{s_i} = X_i$, $Y_{s_i} = 0$, $X_r = X_{M+1}$, $Y_r = Y_{M+1}$, $X_d = 0$, and $Y_d = Y_{M+2}$. Hence, this theorem is a direct application of theorem 4. □

2.5.4 Multiple access multiple relay channel capacity upper bound

In Multiple Access Multiple Relay Channel (MAMRC) (fig. 2.7), a set of sources $\mathcal{S} = \{s_1, s_2, \ldots, s_M\}$ want to send their messages to a common destination d with a help of a set of relays $\mathcal{R} = \{r_1, r_2, \ldots, r_L\}$.

Theorem (7) *If the information rates $\{R_s\}, \forall s \in \mathcal{S}$ are achievable, there exists some joint probability distribution $p(x_\mathcal{S}, x_\mathcal{R})$ such that*

$$\sum_{s \in \mathcal{U}} R_s \leq \min_{\mathcal{T} \subseteq \mathcal{R}} I(X_\mathcal{U}, X_\mathcal{T}; Y_{\mathcal{T}^c}, Y_d | X_{\mathcal{T}^c}, X_{\mathcal{U}^c}) \quad \textit{for all} \quad \mathcal{U} \subseteq \mathcal{S}. \tag{2.51}$$

Proof. The MAMRC is a special case of the multiterminal network with $N = M + L + 1$, $X_{s_i} = X_i$, $Y_{s_i} = 0$, $X_{r_j} = X_{M+j}$, $Y_{r_j} = Y_{M+j}$, $X_d = 0$, and $Y_d = Y_{M+L+1}$. Hence, this theorem is a direct application of theorem 4. $\square$

CHAPTER 3

Static Selective Relaying for Slow-Fading Multiple-Access Multiple-Relay Channels

In this chapter, we study the benefit of Joint Network Channel Coding (JNCC) and Decoding (JNCD) for half-duplex slow fading Multiple-Access Multiple-Relay Channels (MAMRC), defined as follows: (1) Multiple statistically independent sources communicate with a single destination with the help of multiple relays; (2) Each relay is half-duplex; (3) The links between the different nodes are subject to slow fading and additive white Gaussian noise; (4) Some links interfere. The MAMRC access schemes differ in the assignment of the available channel uses to the sources and the relays, ranging from the less efficient orthogonal assignment where sources and relays are given non-overlapping channel uses to the most efficient one where sources and relays are allowed to transmit simultaneously and interfere in all the transmission cycle. Based on the above definition of MAMRC, an information outage analysis of a general access schemes that capture the different possible MAMRC access schemes, is conducted in JNCC/JNCD framework and Distributed Channel Coding/Joint Distributed Channel Decoding (DCC/JDCD). Then, practical JNC coding schemes, at the relays, are proposed together with JNC decoding receiver architectures, at the relays and the destination. The receivers of the relays and the destination are designed to benefit from the signals of the previously activated relays to better decode the sources. The proposed coding and decoding schemes are shown to perform close to the theoretical limit in a variety of simulation scenarios.

3.1 System model

We consider a set $\mathcal{S} = \{s_1, \ldots, s_M\}$ of statistically independent sources. Each source $s \in \mathcal{S}$ wants to communicate its packet $\mathbf{u}_s$ of K_s information bits to a single destination d with the help of a set $\mathcal{R} = \{r_1, \ldots, r_L\}$ of relays (see fig. 3.1). The relays are half-duplex, the half-duplex constraint being implemented in the time domain. A direct path exists from each source to the destination. There is no channel state information at the senders (sources or relays). $\mathcal{N} = \mathcal{S} \cup \mathcal{R} \cup \{d\}$ denotes the set of all nodes in the network. There is no feedback channel between the nodes. Each source s is equipped with one transmit antenna. Each relay is equipped one transmit antenna and $m_r \geq 1$ receive antennas. The destination is equipped with m_d receive antennas. The overall transmission spreads over N channel uses, so that the spectral efficiency of the source s is $R_s = K_s/N$. The N channel uses are divided into T consecutive time slots with $T \in \{2, \ldots, M + L\}$. The number of channel uses in time slot i is denoted by N_i under the constraints $N_i \in \mathbb{N}$ and $N = \sum_{i=1}^{T} N_i$. At a given time instant, a sender $a \in \mathcal{N} \backslash \{d\}$ is said to belong to $\mathcal{R}$ if it tries to forward a helpful signal for the other nodes. Otherwise, it belongs to $\mathcal{S}$. During the time slot i, a set of nodes, denoted by $\mathcal{T}_i \subseteq \mathcal{S} \cup \mathcal{R}$, are allowed to transmit simultaneously. Perfect synchronization is assumed. It is understood that the relays need to listen to the sources in at least one time slot, meaning that $\mathcal{T}_1 \subseteq \mathcal{S}$. The transmitted sequence $\mathbf{x}_s$ of the source s may be partitioned and may spread over multiple time slots. At the beginning of time slot $i \in \{2, \ldots, T\}$, each relay $r \in \mathcal{T}_i$ decodes the sources' packets on the basis of the received signals from the sources and the other previously activated relays. Let $\mathcal{S}_r \subseteq \mathcal{S}$ denote the set of sources that the relay r can decode without errors, referred to as decoding set. If $\mathcal{S}_r = \emptyset$, the relay r remains idle. Otherwise, the relay r generates a network coded packet $\mathbf{u}_r$ of K_r bits, as a function F_r of $\underline{\mathbf{u}}_{\mathcal{S}_r}$, which is subsequently channel encoded. Only a fraction of the coded bits is retained for transmission and here lies the main difference between SNCC and JNCC, which also calls for JNCD. Contrary to the sources, a relay r is not allowed to transmit in more than one time slot, and the time slot in which r may transmit has index $t_r \in \{2, \ldots, T\}$. The sequence transmitted by the source s (resp. relay r) in the time slot i is denoted by $\mathbf{x}_{s,i}$ (resp. $\mathbf{x}_{r,i}$). The discrete-time baseband equivalent signal at the receiver $b \in \mathcal{N} \backslash \mathcal{T}_i$, in time slot $i \in \{1, \ldots, T\}$, is

$$\mathbf{y}_{b,i,k} = \sum_{a \in \mathcal{T}_i} \sqrt{\gamma_{a,b}} \mathbf{h}_{a,b} x_{a,i,k} + \mathbf{n}_{b,i,k}, \quad \forall k \in \{1, \ldots, N_i\},, \tag{3.1}$$

where $x_{a,i,k}$ belongs to the signal set $\mathcal{X}_a$, $\gamma_{a,b}$ is the average Signal-to-Noise Ratio (SNR) for the link (a, b), taking into account path loss and shadowing, $\mathbf{h}_{a,b} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$ is the channel gain vector of the link (a, b) due to nonselective multipath fading (that stays constant during the whole transmission, but changes independently from one transmission to the next), and $\mathbf{n}_{b,i,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$ is the additive white Gaussian noise (AWGN) vector. In the sequel, $\boldsymbol{\gamma}$ and $\mathbf{h}$ respectively denote the set of average SNRs and channel gain vectors

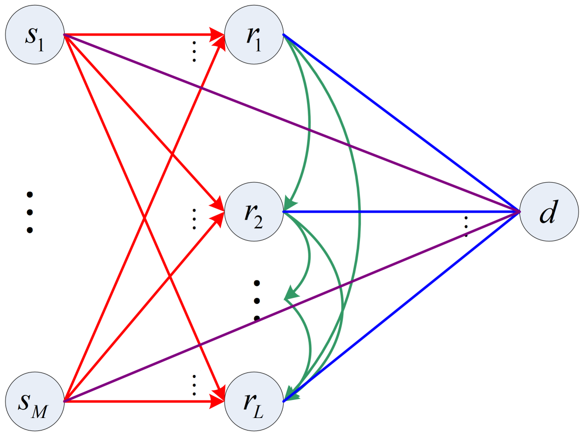


Figure 3.1: The $(M, L, 1)$-MAMRC

for all possible pairs (a, b). The channel gain vectors are assumed mutually independent. $\mathbf{Y}_{b,i}$ denotes the collection of received samples at b during N_i channel uses, $\mathbf{Y}_d$ the collection of received samples at d during N channel uses, and $\mathbf{Y}_r$ the collection of received samples at r during $N_r = \sum_{i=1}^{t_r-1} N_i$ channel uses. To let the receivers (destination, and for some MA schemes, relays) know which relay is there, cooperating, and which sources' packets are included in the relay's transmitted signal, a side information of M bits is required. All receivers implement JNCD, assuming perfect channel (and side) information.

3.1.1 Examples of MAMRC

MA schemes differ depending on how channel uses are allocated to the senders (sources and relays), ranging from the less spectrally-efficient Orthogonal Multiple-Access (OMA) scheme, where senders transmit in non-overlapping channel uses, to the most spectrally-efficient efficient Non-Orthogonal Multiple-Access (NOMA) scheme, where senders are allowed to transmit simultaneously and interfere. Different examples are depicted in Fig. 3.2.

Orthogonal MAMRC (OMARC). During the first phase of αN channel uses, $\alpha \in]0, 1]$ being the cooperation level, the sources transmit orthogonally and the relays listen. During the second phase of $\bar{\alpha}N$ channel uses with $\bar{\alpha} = 1 - \alpha$, the sources remain idle and the relays transmit orthogonally. Hence, $T = M + L$. For $i \in \{1, \ldots, M\}$, $\mathcal{T}_i = \{s_i\}$ and e.g., $N_i = \alpha N/M$. For $j \in \{1, \ldots, L\}$, $\mathcal{T}_{M+j} = \{r_j\}$ and e.g., $N_{M+j} = \bar{\alpha}N/L$.

Semi-Orthogonal MAMRC type I (SOMAMRC-I) [81,93,96]. During the first phase of αN channel uses, the sources transmit simultaneously while the relays listen. During the second phase of $\bar{\alpha}N$ channel uses, the sources stay idle while the relays transmit simultaneously. Hence, $T = 2$, $\mathcal{T}_1 = \mathcal{S}$, $N_1 = \alpha N$, $\mathcal{T}_2 = \mathcal{R}_a$ where $\mathcal{R}_a = \{r \in \mathcal{R} : \mathcal{S}_r \neq \emptyset\}$ is the set of active (i.e., cooperating) relays, and $N_2 = \bar{\alpha}N$.

Semi-Orthogonal MAMRC type II (SOMAMRC-II) [93]. During the first phase of αN channel uses, each source transmits orthogonally. During the second phase of $\bar{\alpha}N$ channel uses, the relays and the sources transmit simultaneously. In this case, $T = M + 1$. For $i \in \{1, \ldots, M\}$, $\mathcal{T}_i = \{s_i\}$ and $N_i = \alpha N/M$. $\mathcal{T}_{M+1} = \mathcal{S} \cup \mathcal{R}_a$ and $N_{M+1} = \bar{\alpha}N$. The main interest of this MA scheme is to decrease the computational complexity at the relays.

Non-Orthogonal MAMRC (NOMAMRC) [94, 126]. During the first phase of αN channel uses, the sources transmit simultaneously. During the second phase of $\bar{\alpha}N$ channel uses, the sources continue transmitting together with the relays. In this case, $T = 2$, $\mathcal{T}_1 = \mathcal{S}$, $N_1 = \alpha N$, and $\mathcal{T}_2 = \mathcal{S} \cup \mathcal{R}_a$, $N_2 = \bar{\alpha}N$. This is the only scenario where we can imagine that the sources are *oblivious* of the presence of the relays.

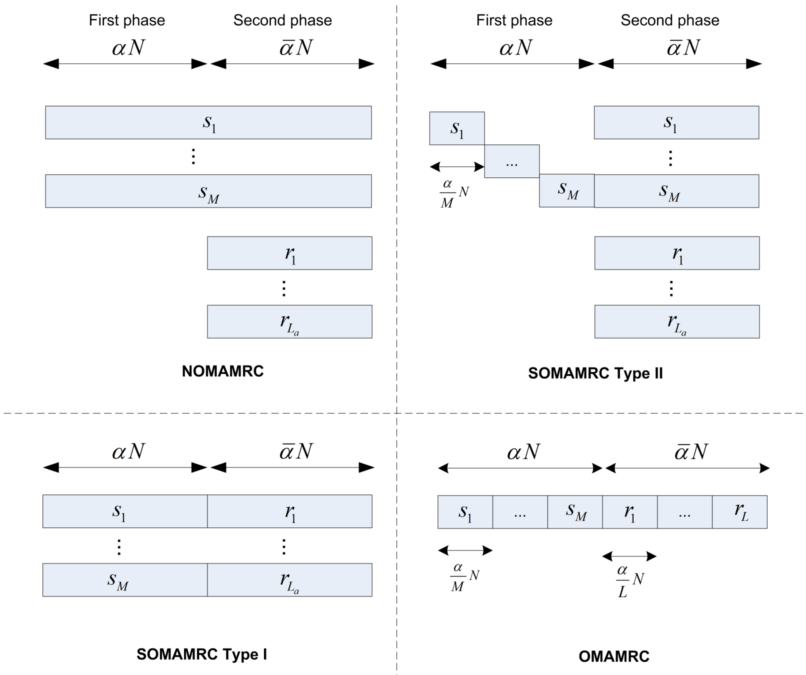


Figure 3.2: Considered channel models: NOMAMRC, SOMAMRC-I, SOMAMRC-II, and OMAMRC

3.2 Outage probability analysis

In this Section, we assume classically that $N_i \to \infty$, where $i = 1, \ldots, T$, and that all the transmitted sequence of the nodes $a \in \mathcal{S} \cup \mathcal{R}$ are i.i.d $\sim \mathcal{CN}(0,1)$ such that the Asymptotic Equipartition Property (AEP) holds (Chapter 3, [123]). In JNCC/JNCD framework, $\mathbf{x}_r \in \mathbb{C}^{N_r}$, the transmitted sequence of a relay r, is a function of the correctly decoded packets of the sources $\underline{\mathbf{u}}_{\mathcal{S}_r}$, i.e., $\mathbf{x}_r(\underline{\mathbf{u}}_{\mathcal{S}_r})$, and forms with $\mathbf{x}_s$, where $s \in \mathcal{S}_r$, a joint codeword for the packets $\underline{\mathbf{u}}_{\mathcal{S}_r}$. In DCC/JDCD framework, no network coding is performed, the transmitted sequence of an active relay is a concatenation of separate sequences, each one corresponds to a correctly decoded packet and denoted by $\mathbf{x}_{r,s}(\mathbf{u}_s) \in \mathbb{C}^{N_r/|\mathcal{S}_r|}$, where $s \in \mathcal{S}_r$. The transmitted sequences $\mathbf{x}_{r,s}(\mathbf{u}_s)$, and $\mathbf{x}_s(\mathbf{u}_s)$ form a joint codeword for the packet $\mathbf{u}_s$ and should be decoded jointly. Note that in separate decoding framework, each one of the sequences $\mathbf{x}_{r,s}(\mathbf{u}_s)$, or $\mathbf{x}_s(\mathbf{u}_s)$ should be a valid codeword for the packet $\mathbf{u}_s$ to be separately decoded. For the sake of notation simplicity, we remove the channel state from the outage event definitions and mutual information expressions in the following. The individual outage event of source s at a receiving node $b \in \{r, d\}$ is denoted by $\mathtt{O}_{b,s}$. The common outage event, which is defined as the event of having at least one source in outage, at node b is denoted by $\mathtt{E}_b$. Given a relay r with $i = ind(r)$, we define the set $\mathcal{R}_i$ as the set of relays with indices less than i, with the convention $\mathcal{R}_1 = \{\emptyset\}$.

We start the outage analysis by OMAMRC since it represents the simplest access scheme. Then we extend the derivation the most general case.

3.2.1 Orthogonal MAMRC outage analysis

In this section, We derive the individual and common outage event of slow fading $(M, L, 1)$-OMAMRC in JNCC/JNCD and DCC/JDCD frameworks, conditional on the channel states $\mathbf{h}_{a,b}$, where $a \in \{s, r\}$, and $b \in \{r, d\}$, and the fixed individual transmission rate R.

3.2.1.1 $(M, L, 1)$-OMAMRC DCC/JDCD outage events

In DCC/JDCD, Each active relay r splits the instantaneous mutual informations, which are exchanged over the R-D and R-R links, equally among the sources of $\mathcal{S}_r$. Using the previous fact and conditional on the decoding set of each relay, we can formulate the individual outage event at the destination as

$$\mathtt{O}_{s,d} = \{R > \frac{\alpha}{M} C_{s,d} + \sum_{\tilde{r} \in \mathcal{R}} \frac{\bar{\alpha}}{L|\mathcal{S}_{\tilde{r}}|} C_{\tilde{r},d} \mathbb{1}_{\{s \in \mathcal{S}_{\tilde{r}}\}}\}, \tag{3.2}$$

and the common outage event as

$$\mathtt{E}_d = \bigcup_{s \in \mathcal{S}} \mathtt{O}_{s,d} = \{R > \min_{s \in \mathcal{S}} \left(\frac{\alpha}{M} C_{s,d} + \sum_{\tilde{r} \in \mathcal{R}} \frac{\bar{\alpha}}{L|\mathcal{S}_{\tilde{r}}|} C_{\tilde{r},d} \mathbb{1}_{\{s \in \mathcal{S}_{\tilde{r}}\}} \right)\}, \tag{3.3}$$

where $C_{a,b}$ is as defined in (2.37), $a \in \{s, r\}$, $b \in \{d, r\}$. In (3.2) and (3.3), the individual and the common outage events at each relay r are obtained by replacing the index d by r and the set $\mathcal{R}$ by $\mathcal{R}_i$, where $i = ind(r)$. Once the individual outage events at the relay r are acquired, the decoding set $\mathcal{S}_r$ will be determined.

3.2.1.2 $(M, L, 1)$-OMAMRC JNCC/JNCD outage events

In JNCC/JNCD, the received signals from the M sources and the L relays will be treated jointly at the destination to better decode the sources' packets. Based on the previous fact, the common outage event at the destination can be written as (see [75] for the special case $(2, 1, 1)$-OMAMRC, and perfect S-R, i.e., $\mathcal{S}_r = \mathcal{S}$)

$$\mathtt{E}_d = \{R > \min_{\mathcal{U} \subseteq \mathcal{S}} \frac{\frac{\alpha}{M} \sum_{s \in \mathcal{U}} C_{s,d} + \sum_{\tilde{r} \in \mathcal{R}} \frac{\bar{\alpha}}{L} C_{\tilde{r},d} \mathbb{1}_{\{\mathcal{S}_{\tilde{r}} \cap \mathcal{U} \neq \emptyset\}}}{|\mathcal{U}|}\}. \tag{3.4}$$

The individual outage event at the destination is given by (see [93, proposition 1])

$$\mathtt{O}_{d,s} = \{R > \max_{\mathcal{I} \subset \mathcal{S}} \min_{\mathcal{U} \subseteq \mathcal{I}^c : s \in \mathcal{U}} \frac{\frac{\alpha}{M} \sum_{\tilde{s} \in \mathcal{U}} C_{\tilde{s},d} + \sum_{\tilde{r} \in \mathcal{R}_s} \frac{\bar{\alpha}}{L} C_{\tilde{r},d}}{|\mathcal{U}|}\}, \tag{3.5}$$

where $\mathcal{R}_s \triangleq \{r \in \mathcal{R} : \mathcal{I} \cap \mathcal{S}_r = \emptyset \wedge s \in \mathcal{S}_r \cap \mathcal{U}\}$ is the set of relays who can be jointly decoded with the source s. The individual and the common outage events at a relay r can be obtained from (3.5), (3.4), and the definition of $\mathcal{R}_s$, by replacing the index d by r and the set $\mathcal{R}$ by $\mathcal{R}_i$, where $i = ind(r)$.

3.2.2 MAMRC outage analysis

In this section, we derive the individual and common outage event of slow fading $(M, L, 1)$-MAMRC in JNCC/JNCD in the generalized access scheme as described in sec. 3.1.

3.2.2.1 (M,L,1)-MAMRC JNCC/JNCD outage events

In JNCC/JNCD, the received signals from the M sources and the L relays, during the T time slots, will be treated jointly to better decode the sources' packets. In order to decode all the sources correctly, the transmission rate R_s of each source s, conditioned on $\{\mathbf{h}_{a,b}\}$, where $a \in \{s, r\}$, and $b \in \{r, d\}$. and the input distribution $\prod_{s\in\mathcal{S}} p(x_a)$, must satisfies

$$R_{\mathcal{U}} \leq \sum_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_k}) \text{ for all } \mathcal{U} \subseteq \mathcal{S}, \tag{3.6}$$

where $\mathcal{U}^c = \mathcal{S} \setminus \mathcal{U}$, $R_{\mathcal{U}} = \sum_{s\in\mathcal{U}} R_s$, $\alpha_i = N_i/N$, $\mathcal{R}_k \triangleq \{r \in \mathcal{R} : \mathcal{S}_r \neq \emptyset \wedge \mathcal{S}_r \subseteq \mathcal{U}^c\}$ is the set of active relays whose signals are perfectly known, and $\mathcal{R}_u \triangleq \mathcal{R}\backslash\mathcal{R}_k$ is the set of relays whose signals are to be jointly decoded with the sources in $\mathcal{U}$. Using (3.6), the common outage event at d can be expressed as

$$\mathtt{E}_d = \{R_{\mathcal{U}} > \sum_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_k}) \text{ for some } \mathcal{U} \subseteq \mathcal{S}\}, \tag{3.7}$$

or equivalently,

$$\mathtt{E}_d = \bigcup_{\mathcal{U}\subseteq\mathcal{S}} \mathtt{F}_{d,\mathcal{S}}(\mathcal{U}), \tag{3.8}$$

where $\mathtt{F}_{d,\mathcal{S}}(\mathcal{U}) = \{R_{\mathcal{U}} > \sum_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_k})\}$ is defined as the event that the sources in $\mathcal{U}$ are in outage (decoded erroneously) assuming that the sources in $\mathcal{U}^c = \mathcal{S} \setminus \mathcal{U}$ are perfectly known (their packets are perfectly known). If the event $\mathtt{F}_{d,\mathcal{S}}(\mathcal{U})$ holds for some $\mathcal{U}$, then d cannot correctly decode the sources in $\mathcal{U}$ knowing perfectly the sources in $\mathcal{U}^c$. In this case, a common outage event for the sources in $\mathcal{S}$ is declared at d. The fact that the event $\mathtt{E}_d$ holds does not mean that d cannot correctly decode a subset of sources in $\mathcal{S}$. Consider $\mathcal{I} \subseteq \mathcal{S}$ the subset of sources which cannot be correctly decoded (considered as interference) and $\mathcal{I}^c = \mathcal{S}\backslash\mathcal{I}$. Let $\mathtt{E}_{d,\mathcal{I}^c}$ denote the common outage event of the $(|\mathcal{I}^c|, L, 1)$-MAMRC at d. We can express this event as

$$\mathtt{E}_{d,\mathcal{I}^c} = \bigcup_{\mathcal{L}\subseteq\mathcal{I}^c} \mathtt{F}_{d,\mathcal{I}^c}(\mathcal{L}), \tag{3.9}$$

where in $\mathtt{F}_{d,\mathcal{I}^c}(\mathcal{L})$, the set of sources in $\mathcal{I}$ are considered as interference which will cause some of the relays signals to be interference as well. Hence, we can express this event as

$$\mathtt{F}_{d,\mathcal{I}^c}(\mathcal{L}) = \{R_{\mathcal{L}} > \sum_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_k})\}, \tag{3.10}$$

$\mathcal{L}^c = \mathcal{I}^c\backslash\mathcal{L}$, i.e, the sources in $\mathcal{L}^c$ are supposed perfectly known , $\mathcal{R}'_k \triangleq \{r \in \mathcal{R} : \mathcal{S}_r \neq \emptyset \wedge \mathcal{S}_r \subseteq \mathcal{L}^c\}$ is the set of active relays whose signals are perfectly known, $\mathcal{R}'_I \triangleq \{r \in \mathcal{R} : \mathcal{S}_r \neq \emptyset \wedge \mathcal{I} \cap \mathcal{S}_r \neq \emptyset\}$ the set of relays whose signals are interference (corrupted by the sources that are considered as interference), and $\mathcal{R}'_u \triangleq \mathcal{R}\backslash\mathcal{R}'_k\backslash\mathcal{R}'_I$ is the set of relays whose signals are to be jointly decoded with the sources in $\mathcal{L}$. Note that equation (3.9) could be seen as special case of (3.7) when $\mathcal{I} = \emptyset$ and in this case $\mathcal{R}'_I = \emptyset$, $\mathcal{R}'_u$ is equivalent to $\mathcal{R}_u$, and $\mathcal{R}'_k$ is equivalent to $\mathcal{R}_k$).

Proposition (3) *The source s is in outage at d if and only if the $(M, L, 1)$-MAMRC is in outage at d and all the $(|\mathcal{I}^c|, L, 1)$-MAMRC where $s \in \mathcal{I}^c$ and the sources in $\mathcal{I}$ are interference are in outage at d. Hence, the individual outage event of s at d is given by*

$$\mathtt{O}_{d,s} = \bigcap_{\mathcal{I}\subset\mathcal{S}:s\in\mathcal{I}^c} \mathtt{E}_{d,\mathcal{I}^c} \tag{3.11}$$

Proof. The sufficient part: If all $(|\mathcal{I}^c|, L, 1)$-MAMRC where the source $s \in \mathcal{I}^c$ are in outage, then s cannot be correctly decoded by any means and is in outage. The necessary part: By contradiction. Suppose that s is in outage and one $(|\mathcal{I}^c|, L, 1)$-MAMRC where $s \in \mathcal{I}^c$ is not in outage. Then, it means that d can jointly decode the sources of this $(|\mathcal{I}^c|, L, 1)$-MAMRC. As a result, the destination can correctly decode s which contradicts the statement that s is in outage. □

The individual outage probability of a particular source s at d and the common outage probability at d are defined as

$$P_{out,d,s}^{ind}(\mathbf{R},\boldsymbol{\gamma},\boldsymbol{\alpha}) = \Pr(\mathtt{O}_{\mathbf{d,s}}) = \mathbb{E}\left[\mathbb{1}_{\{\mathtt{O}_{\mathbf{d,s}}\}}\right] = \int_{\mathbf{h}} \mathbb{1}_{\{\mathtt{O}_{\mathbf{d,s}}(\mathbf{h})\}}\mathbf{p}(\mathbf{h})\mathbf{d}(\mathbf{h}), \tag{3.12}$$

and

$$P_{out,d}^{com}(\mathbf{R},\boldsymbol{\gamma},\boldsymbol{\alpha}) = \Pr(\mathtt{E}_{\mathbf{d}}) = \mathbb{E}\left[\mathbb{1}_{\{\mathtt{E}_{\mathbf{d}}\}}\right] = \int_{\mathbf{h}} \mathbb{1}_{\{\mathtt{E}_{\mathbf{d}}(\mathbf{h})\}}\mathbf{p}(\mathbf{h})\mathbf{d}(\mathbf{h}). \tag{3.13}$$

respectively, where $\boldsymbol{\alpha}$ is the vector of the values α_i's, which can be represented by one value for the MAMRC defined in Section 3.1.1. And $\mathbf{R}$ is the vector of $R_s, \forall s \in \mathcal{S}$.

For i.i.d Gaussian input distributions, i.e., $p(x_a) \sim \mathcal{CN}(0,1)$, where $a \in \{s, r\}$, the instantaneous mutual information expressions used in the previous outage events can be expressed as

$$I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_k}) = \log\left(1 + \frac{\sum\limits_{a\in\mathcal{T}_i\cap(\mathcal{L}\cup\mathcal{R}'_u)} \gamma_{a,d}||\mathbf{h}_{a,d}||^2}{1 + \sum\limits_{a\in\mathcal{T}_i\cap(\mathcal{I}\cup\mathcal{R}'_I)} \gamma_{a,d}||\mathbf{h}_{a,d}||^2}\right) \tag{3.14}$$

Finally, the individual/common outage events at a relay r can be obtained from (3.15) and (3.16) by replacing d by r, $\mathbf{y}_{d,i}$ by $\mathbf{y}_{r,i}$, and T by $t_r - 1$.

3.2.3 Symmetric assumptions

A symmetric system contains sources with same priority and with symmetric S-R, S-D links. The R-R and R-D links could be symmetric or asymmetric, but for simplicity we assume them symmetric as well.

By assuming sources with same priority, i.e., transmit with the same rate $R_s = R, \forall s \in \mathcal{S}$, the common outage event at d, can be written as

$$\mathtt{E}_d = \left\{ R > \min_{\mathcal{U}\subseteq\mathcal{S}} \frac{\sum\limits_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_u}; \mathbf{y}_{d,i}|\mathbf{x}_{\mathcal{T}_i\cap\mathcal{U}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}_k})}{|\mathcal{U}|} \right\}, \tag{3.15}$$

and the individual outage event of s at d could be written as

$$\mathtt{O}_{d,s} = \left\{ R > \max_{\mathcal{I}\subset\mathcal{S}} \min_{\mathcal{L}\subseteq\mathcal{I}^c : s\in\mathcal{L}} \frac{\sum_{i=1}^{T} \alpha_i I(\mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_u}; \mathbf{y}_{d,i} | \mathbf{x}_{\mathcal{T}_i\cap\mathcal{L}^c}, \mathbf{x}_{\mathcal{T}_i\cap\mathcal{R}'_k})}{|\mathcal{U}|} \right\}. \tag{3.16}$$

By going one step further in the symmetry assumption we introduce $\gamma_{s,b} = c_{s_1,b}\gamma$, $\forall s \in \mathcal{S}$, and $\gamma_{r,b} = c_{r_1,b}\gamma$, $\forall r \in \mathcal{R}$, where $b \in \mathcal{R} \cup \{d\}$, and $c_{s_1,b}, c_{r_1,b} \in \mathbb{R}^+$ are finite constants, accounting for large scale path loss, and shadowing effects. γ is the transmit power to received noise ratio. Due to the symmetric rate and links assumptions in the system, we can drop index s in the symetric individual outage probability and we define the symmetric diversity order at d as follow

$$D_d \triangleq \lim_{\gamma\to\infty} \frac{-\log\left(P^{ind}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha})\right)}{\log(\gamma)} \overset{(a)}{=} \lim_{\gamma\to\infty} \frac{-\log\left(P^{com}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha})\right)}{\log(\gamma)}. \tag{3.17}$$

where (a) follows by the fact that $P^{ind}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha}) \leq P^{com}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha}) \leq MP^{ind}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha})$.

Proposition (4) *The symmetric diversity order of $(M, L, 1)$-MAMRC is given by*

$$D_d = \min(m_r, m_d)L + m_d \tag{3.18}$$

Proof. The proof is given in Appendix A. □

Based on (3.12) and (3.13), the symmetric individual and common ϵ-outage achievable rates are defined as

$$R^{ind}_{\epsilon}(\boldsymbol{\gamma},\boldsymbol{\alpha}) = sup\{R : P^{ind}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha}) \leq \epsilon\} \tag{3.19}$$

and

$$R^{com}_{\epsilon}(\boldsymbol{\gamma},\boldsymbol{\alpha}) = sup\{R : P^{com}_{out,d}(R,\boldsymbol{\gamma},\boldsymbol{\alpha}) \leq \epsilon\}, \tag{3.20}$$

respectively. Finding closed-form analytical expressions of the individual and common outage probabilities (3.12) and (3.13) is clearly a difficult task. It is already not trivial for a simple M-sender MAC with $M \geq 3$ (see [48] for the 2-sender MAC). This topic is left out of the scope of the paper, and the expressions are instead evaluated numerically (Monte-Carlo integration).

In the sequel, when there is no ambiguity we might drop some indexes or remove some parameters that the achievable rates and outage probability depends on.

3.2.4 Numerical results

We consider Gaussian i.i.d inputs, where the instantaneous mutual information are computed using (3.14). There are, of course, an infinity of MAMRC configurations (SNR distribution, transmission rate, number of sources, number of relays, etc.). By carefully selecting a few configurations as examples, our intention is to better understand how the different coding strategies perform when different access schemes are used. We start by evaluating the probability of individual outage events and we show the symmetric ϵ-outage achievable rates.

3.2.4.1 OMAMRC numerical results

In this section, we focus on OMAMRC where the superiority of JNCC/JNCD over DCC/JDCD will be shown.

Comparison of JNCC/JNCD and DCC/JDCD We compare the individual outage probability $p_{out}^{ind}(\gamma_{r,d}, \gamma_{s,r}, \gamma_{s,d}, \gamma_{r,\tilde{r}})$ of JNCC/JNCD and DCC/JDCD. First, we consider a $(2,2,1)$-OMAMRC with no R-R links ($\gamma_{r,\tilde{r}} = 0$), $R = 1/3$, and $\alpha = 2/3$. The corresponding results are depicted in Fig. 3.3 for $m_d = 1, 4$. It can be seen that: (1) Both JNCC/JNCD and DCC/JDCD schemes achieve the full diversity given by proposition 4 (2) DCC/JDCD requires a higher SNR to achieve the full diversity; (3) JNCC/JNCD has a better coding gain than DCC/JDCD; (4) When $m_d = 4$ and lossy S-R channels, there is no difference between both schemes, because the relay is in outage with probability close to one at this range of SNR (we need to increase the quality of S-R links in order to see the difference). Next, we consider $(4,2,1)$-MAMRC, with $m_d = 4$, $R = 1/5$ and $\alpha = 4/5$. We increase the quality of S-R links by 10 dB with respect to the previous simulation, and we chose $\gamma_{r,\tilde{r}} = 10\gamma$. The corresponding results are depicted in Fig. 3.4. The same previous observations still hold except the last one.

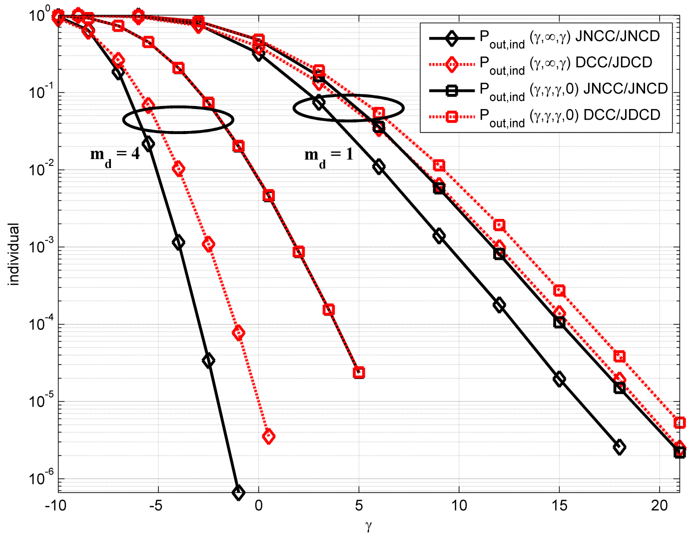


Figure 3.3: P_{out}^{ind} of JNCC/JNCD vs DCC/JDCD for $(2, 2, 1)$-OMAMRC, where $R = 1/3$, and $\alpha = 2/3$.

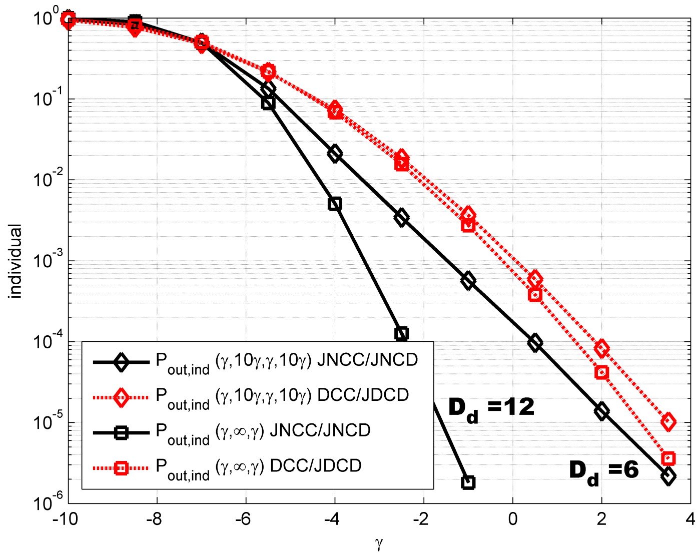


Figure 3.4: P_{out}^{ind} of JNCC/JNCD vs DCC/JDCD for $(4, 2, 1)$-OMAMRC, where $m_d = 4, m_r = 1$, $R = 0.2$, and $\alpha = 0.8$.

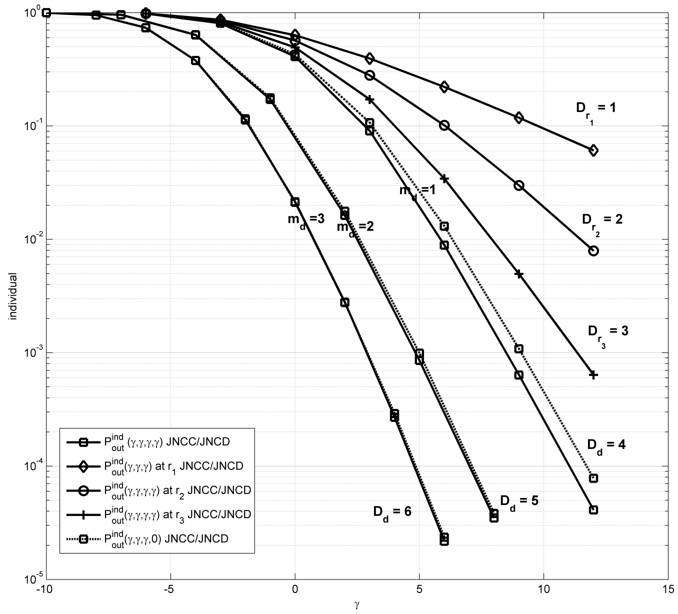


Figure 3.5: P_{out}^{ind} of JNCC/JNCD at the relays and the destination of $(3, 3, 1)$-OMAMRC, where $m_d = 1, 2, 3, m_r = 1$, $R = 2/9, \alpha = 2/3$

Inter-relays links and diversity order We only consider the JNCC/JNCD scheme. We choose $(3, 3, 1)$-OMAMRC and show the individual outage probability of each relay and the destination, where $R = 2/9$, $\alpha = 2/3$, and $m_d = 1, 2, 3$, both cases are considered $\gamma_{r,\tilde{r}} = 0$ and $\gamma_{r,\tilde{r}} = \gamma$. Fig. 3.5 shows the corresponding results. As we can see, inter-relay communication increases the diversity order at the relays but it does not increase the diversity order at the destination. which means that inter-relays communication can increase the coding gain but not the diversity order at the destination, this behavior was expected from the proof of proposition 4. The amount of coding gain can vary according to the investigated scenario.

3.2.4.2 MAMRC numerical results

In this section, we compare the performance of the different access schemes described in section 3.1.1 when JNCC/JNCD is used.

Individual outage probability We Consider $(2, 2, 1)$-MAMRC and $(2, 4, 1)$-MAMRC, we consider $\alpha = 2/3$, and $R = 2/3$ (b./c.u) (these values correspond to the coding modulation scheme that will be used in section 3.3.6). The number of receive antennas at the destination $m_d \in \{1, 4\}$. In NOMAMRC, when $m_d = 1$, we take $\gamma_{s,d} = \gamma_{s,r} = \gamma_{r,d} = \gamma$, and when $m_d = 4$, we increase the reliability of the S-R links by 20 dB hence, $\gamma_{s,d} = \gamma_{r,d} = \gamma$, and $\gamma_{s,r} = 100\gamma$. For the comparison to be fair with other access schemes, the energy budget per source (per available dimensions) should be always the same. In SOMAMRC-I, when $m_d = 1$ we chose $\gamma_{s,d} = \gamma/\alpha$, and $\gamma_{s,r} = \gamma_{r,d} = \gamma$, and when $m_d = 4$, we take $\gamma_{s,d} = \gamma/\alpha$, $\gamma_{r,d} = \gamma$, and $\gamma_{s,r} = 100\gamma$. In OMAMRC, when $m_d = 1$, we chose $\gamma_{s,d} = M\gamma/\alpha$, $\gamma_{r,d} = L\gamma$, and $\gamma_{s,r} = M\gamma$, and when $m_d = 4$, we chose $\gamma_{s,d} = M\gamma/\alpha$, $\gamma_{r,d} = L\gamma$, and $\gamma_{s,r} = M100\gamma$. Fig. 3.6, shows the results for NOMAMRC, SOMAMRC-I, OMAMRC, and MAC (a NOMAMRC where all the relays are switched of). As expected, the OMAMRC has the worst performance among the investigated relay-assisted communication schemes. We notice also that all the different investigated MAMRC schemes has the same diversity order, which equals to the one given in proposition 4. Surprisingly SOMAMRC-I has a very close performance, and sometimes slightly better, to NOMAMRC, this can be justify by the relatively low transmission rate that is used in this simulation where the potentials of NOMAMRC have not been exploited. To be sure that SOMAMRC-I can not be as efficient as the NOMAMRC we fix an outage probability to ϵ and calculate the maximum rate which the access schemes can achieve under this constraint.

Individual ϵ-outage achievable rate The symmetric individual ϵ-outage achievable rate R_ϵ^{ind}, computed for $\epsilon = 0.01$ and $\alpha = 2/3$, is shown. We fix the number of relays to $L = 2$ and the number of sources $M = 1, 2, 3, 4$. In order to evaluate the benefit of relay-assisted communication, we also compute the symmetric individual ϵ-outage achievable rate of an M-user MAC considering no relay in the system. The N channel uses are then assigned to the sources to transmit either orthogonally (OMAC) or simultaneously (MAC). we assume that all the sources, relays, and destination are equipped with single receive or/and single transmit antenna, i.e., $m_d = m_r = 1$, For SOMAMRC-I, we chose $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma$. For this comparison to be fair, the energy budget per source (per available dimensions) is always the same. Hence, for OMAMRC, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = M\gamma$, and $\gamma_{s,d} = M\gamma$. For SOMAMRC-II, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = M\gamma$, and $\gamma_{s,d} = \frac{\alpha}{\alpha/M+\bar{\alpha}}\gamma$. For NOMAMRC, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = \gamma$, and $\gamma_{s,d} = \alpha\gamma$. Fig. 3.7 shows R_ϵ^{ind} of the three MAMRC access schemes, described in section 3.1.1, and the OMAMRC. We observe that

- the access schemes have the same achievable rates at low SNR,
- as expected, NOMAMRC has the best achievable rate over all SNR, while OMAMRC

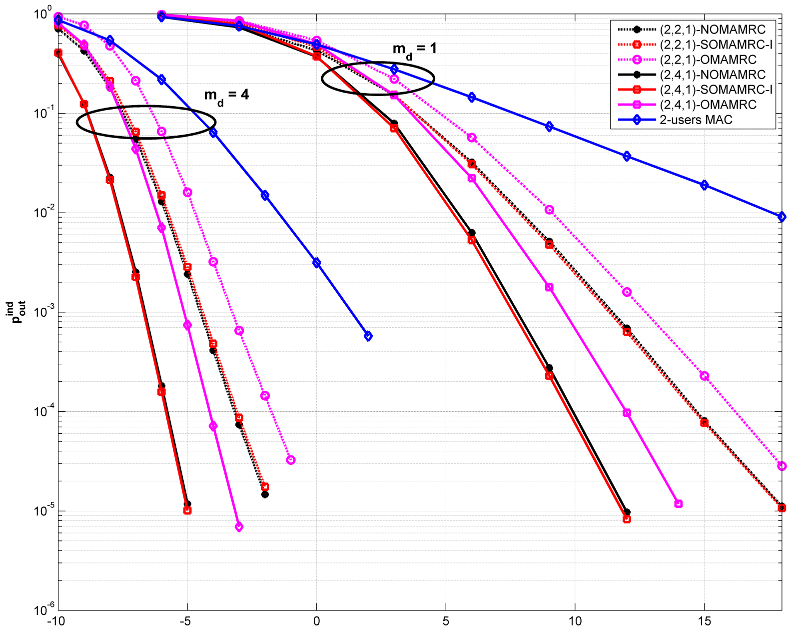


Figure 3.6: p_{out}^{ind} for $(2,2,1)$-MAMRC and $(2,4,1)$-MAMRC different access schemes and 2-users MAC. Where $R = 2/3$ (b./c.u), and $\alpha = 2/3$.

has the worst,

- the achievable rate of SOMAMRC-I is similar to the one of NOMAMRC at moderate SNR, which is no more the case at high SNR where the rate of SOMAMRC-II becomes better,
- at high SNR, the achievable rate of NOMAMRC converges to the one of the MAC. Indeed, in [94], it was proven (for $m_d = m_r = 1$) that, at high SNR, the relays cannot help any more and that the probability that a relay be inactive (or passive) goes to one. All the other schemes have smaller achievable rates than the MAC since the sources in these schemes do not make use of all the available channel uses and the probability that the relays be inactive is high.

3.3 JNC distributed coding and decoding of NOMAMRC

In this section, we explain when and how the JNCC is performed, detail the structure of the encoders, and provide an complete algorithmic description of the JNCD based on factor

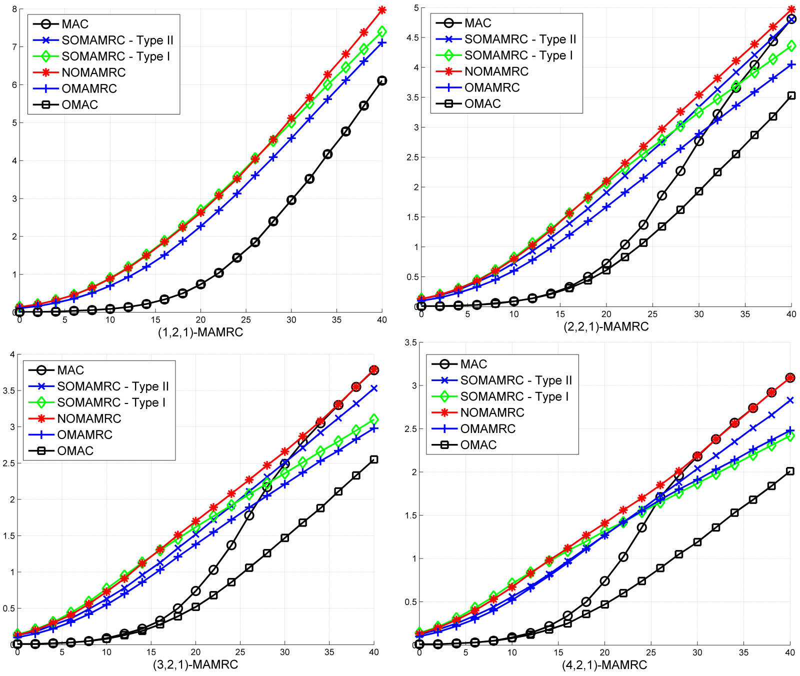


Figure 3.7: $R_{\epsilon}^{ind}(\gamma)$ with $M = 1, 2, 3, 4$, $L = 2$, $n = m_r = m_d = 1$.

graphs and the sum-product algorithm [115]. We intentionally focus on the NOMAMRC, i.e., $T = 2$, $\mathcal{T}_1 = \mathcal{S}$ and $\mathcal{T}_2 = \mathcal{S} \cup \mathcal{R}$, because it is the most complicated scenario. For the other possible access schemes, one can combine the design concepts, which will be presented in this section, with the ones which were presented in [97].

3.3.1 Coding at the sources

The sources' packets (information bits and checks of CRC codes) are binary vectors $\mathbf{u}_s \in \mathbb{F}_2^K$ of length K. Each source basically employs a Bit Interleaved Coded Modulation (BICM) [127]. Binary vectors are first encoded with linear binary codes $C_s : \mathbb{F}_2^K \longrightarrow \mathbb{F}_2^{n_s}$ into binary codewords $\mathbf{c}_s \in \mathbb{F}_2^{n_s}$. For C_s we choose a puncturing systematic turbo codes of coding rate w_s. Each C_s consists of: (1) Two Recursive Systematic Convolutional (RSC) encoders with generator matrix $\mathbf{G}_s(D)$, concatenated in parallel using optimized semi-random interleaver π; (2) A puncturing pattern that is responsible of configuring the rate of the turbo code w_s, and generating the turbo codeword $\mathbf{c}_s \in \mathbb{F}_2^{n_s}$; (3) A puncturing vector, denote by $\mathbf{p}_s \in \mathbb{F}_2^{n_s}$, responsible of selecting witch parts of $\mathbf{c}_s$ will be send in each time slot. For the first transmission phase, the coded bits $\mathbf{c}_s$ are first punctured following the puncturing vector $\mathbf{p}_s$. The resulting bits $\mathbf{c}_{s,1}$ are then bit-interleaved into $\mathbf{b}_{s,1} \in \mathbb{F}_2^{n_{s,1}}$ using $\pi_{s,1}$. Each interleaved binary codeword $\mathbf{b}_{s,1}$ is mapped by a memoryless modulator $\phi_s : \mathbb{F}_2^{q_s} \to \mathcal{X}_s$ to the modulated sequence $\mathbf{x}_{s,1} \in \mathcal{X}_s^{N_1}$. For the second transmission phase, the coded bits that are punctured during the first phase forms $\mathbf{c}_{s,2} \in \mathbb{F}_2^{n_{s,2}}$, where $n_s = n_{s,1} + n_{s,2}$. $\mathbf{c}_{s,2}$ are bit-interleaved into $\mathbf{b}_{s,2} \in \mathbb{F}_2^{n_{s,2}}$ using $\pi_{s,2}$. Then mapped by ϕ_s to the modulated sequence $\mathbf{x}_{s,2} \in \mathcal{X}_s^{N_2}$.

3.3.2 Relaying function

At the end of the first phase, the relays try to decode the packets of the sources, processing the received samples $\mathbf{Y}_{r,1}$ (the processing time is neglected). Joint multiser detection and decoding is performed by means of the sum-product algorithm. Since this part is standard (see e.g., [128]), details are omitted. Based on CRC checks, relays can decide if a source's packet is correctly decoded or not. When a relay $r \in \mathcal{R}$ decides to transmit (i.e., has correctly decoded *at least* one packet), it applies a local network coding function $F_r : \mathbb{F}_2^{K \times M} \to \mathbb{F}_2^K$ to generate its network coded packet. Let $\mathbf{u}_r = F_r(\mathbf{u}_{s_1}, \ldots, \mathbf{u}_{s_M})$ be the relay binary network coded packet of length K. Any incorrectly decoded source's packet will be replaced by $\mathbf{0}_K$. the detailed structure of F_r will be discussed in Sec. 3.3.3. Each cooperating relay employs a BICM. Binary network coded packets are encoded with linear binary codes $C_r : \mathbb{F}_2^K \to \mathbb{F}_2^{n_r}$ into binary codewords $\mathbf{c}_{r,2} \in \mathbb{F}_2^{n_r}$. C_r consists of a rate-1/2 RSC encoder with generator matrix $\mathbf{G}_r(D)$, followed by a puncturing vector $\mathbf{p}_r$ that gives more importance to parity bits than systematic bits. The final coding rate of the punctured code C_r is denoted by w_r. The binary codeword $\mathbf{c}_{r,2}$ is then bit-interleaved into $\mathbf{b}_{r,2} \in \mathbb{F}_2^{n_r}$ using $\pi_{r,2}$. The interleaved binary codeword $\mathbf{b}_{r,2}$ is mapped by a memoryless modulator $\phi_r : \mathbb{F}_2^{q_r} \to \mathcal{X}_r$ to the modulated sequence $\mathbf{x}_{r,2} \in \mathcal{X}_r^{N_2}$. The relation between the different parameters are $R = q_s w_s$ and $\alpha = 1 - \frac{q_s w_s}{q_r w_r}$.

Note 1: To let the destination detect which of the frames are included in a relay's transmitted signal, each relay transmits a side information (M additional bits) to indicate its state. This side information is perfectly known at the destination.

Note 2: The separate functions F_r, which are applied locally at each relay, can be seen as components of a global function denoted by $F_{nc} : \mathbb{F}_2^{K \times M} \to \mathbb{F}_2^{K \times L}$, referred to as global network coding function.

3.3.3 Linear network coding

In GFNC, the network coded packet at the relay $r \in \mathcal{R}$ can be expressed as

$$\mathbf{u}_r = F_r(\mathbf{u}_{s_1}, \ldots, \mathbf{u}_{s_M}) = \psi^{-1}\left(\sum_{s\in\mathcal{S}} \alpha_{s,r} * \psi(\mathbf{u}_s)\right), \tag{3.21}$$

where $\psi : \mathbb{F}_2^K \to \mathbb{F}_{2^q}^{K/q}$ is a function that converts a vector of bits of length K into a vector of $\mathbb{F}_{2^q}$ elements of length K/q ($q \geq 2$), ψ^{-1} its inverse, $\sum$ and $*$ are the sum and multiplication operators in $\mathbb{F}_{2^q}$, and $\alpha_{s,r}$'s are the network coding coefficients in $\mathbb{F}_{2^q}$. Any incorrectly decoded packet in (3.21) will be replaced by the all-zeros vector $\mathbf{0}_K$. Using the binary representation of elements in $\mathbb{F}_{2^q}$, we can equivalently write (3.21) as

$$\mathbf{u}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{G}_{s,r}\mathbf{u}_s \mathbb{1}_{\{s\in\mathcal{S}_r\}}, \tag{3.22}$$

where $\sum\oplus$ represents the sum operator in $\mathbb{F}_2$, and $\mathbf{G}_{s,r}$ is a $K \times K$ binary matrix depends on the chosen coefficients $\alpha_{s,r}$. The matrix $\mathbf{G}_{s,r}$ has the form $\mathbf{G}_{s,r} = \mathbf{I}_{K/q} \otimes \mathbf{G}(\alpha_{s,r})$ where $\mathbf{G}(\alpha_{s,r})$ is a $q \times q$ binary matrix which depends on the network coding coefficient $\alpha_{s,r}$. In BFNC, the network coded packet at the relay $r \in \mathcal{R}$ is given by

$$\mathbf{H}_r\mathbf{u}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{H}_{s,r}\mathbf{u}_s \mathbb{1}_{\{s\in\mathcal{S}_r\}}, \tag{3.23}$$

where $\mathbf{H}_r$, and $\mathbf{H}_{s,r}$, for all $s \in \mathcal{S}$, and $r \in \mathcal{R}$ are $K \times K$ binary matrices. From (3.22) and (3.23), a direct relation between the BFNC and the GFNC is seen, both coding schemes are similar if $\mathbf{H}_r = \mathbf{I}_K$ and $\mathbf{H}_{s,r} = \mathbf{G}_{s,r}$. Thus, code designs for GFNC can be directly translated to code designs for BFNC, but the converse is not true in general.

We now go one step further into the description of the two proposed coding strategies:
1) GFNC with coefficients chosen from MDS codes and randomly interleaved packets: Each relay r generates its network coded packet using the following operation

$$\tilde{\mathbf{u}}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{G}_{s,r}\tilde{\mathbf{u}}_{s,r} \mathbb{1}_{\{s\in\mathcal{S}_r\}}, \tag{3.24}$$

where $\tilde{\mathbf{u}}_{s,r} = \Pi_{s,r}(\mathbf{u}_s)$, and $\tilde{\mathbf{u}}_r = \Pi_r(\mathbf{u}_r)$ are interleaved versions of the packets of the sources and the relay network coded packet, respectively. The interleavers $\Pi_{s,r}, \Pi_r$ are independent pseudo random interleavers of length K. They are used to limit the detrimental effect of short cycles on the performance of the sum-product algorithm. If the network coding coefficients are chosen from $[M+L, M, L+1]$ Reed-Solomon codes, then GFNC achieves the full diversity order.

2) BI-XOR based network coding, initially proposed in [114]: Each relay r generates its network coded packet using (3.24) by replacing $\mathbf{G}_{s,r}$ by $\mathbf{I}_K$. BI-XOR based network coding has the flexibility and simplicity of (linear) random network codes. Furthermore, BI-XOR based network coding can achieve a diversity order close to the full diversity order with high probability [114, Theorem 5].

3.3.4 Joint network channel decoding at the destination

The destination implements JNCD and starts decoding at the end of the second phase by processing the received samples $\mathbf{Y}_{d,1}$ and $\mathbf{Y}_{d,2}$ (see (3.1)), and taking into account the side information transmitted by the cooperating relays. The maximum a posteriori decoding rule is given by

$$\begin{aligned}
\hat{u}_{s,k} &= \arg\max_{u_{s,k}\in\mathbb{F}_2} P(u_{s,k}|\mathbf{Y}_d, B) && \text{(3.25a)}\\
&= \arg\max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} p(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}_a}, \underline{\mathbf{c}}_{\mathcal{S},1}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{b}}_{\mathcal{S},1}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \mathbf{Y}_d)\\
&\times \Xi_B(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}_a}, \underline{\mathbf{c}}_{\mathcal{S},1}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{b}}_{\mathcal{S},1}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2}) && \text{(3.25b)}\\
&\overset{(a)}{=} \arg\max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} p(\mathbf{Y}_{d,1}, \mathbf{Y}_{d,2}|\underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2})\\
&\times \Xi_B(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}_a}, \underline{\mathbf{c}}_{\mathcal{S},1}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{b}}_{\mathcal{S},1}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2}) && \text{(3.25c)}\\
&\overset{(b)}{=} \arg\max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} \left(\prod_{i=1}^{2} p(\mathbf{Y}_{d,i}|\mathbf{x}_{\mathcal{T}_i,i})\right)\\
&\times \Xi_B(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}_a}, \underline{\mathbf{c}}_{\mathcal{S},1}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{b}}_{\mathcal{S},1}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2}) && \text{(3.25d)}
\end{aligned}$$

where Ξ_B is the characteristic function for B, the behavioral modeling of JNCC at the destination that captures the relationship between the different variables of the system. (a) Follows from using the Bayes rule, and the assumption that sources' packets have uniform priors. (b) Follows from the system model assumptions detailed in II.A and II.B. The characteristic function Ξ_B can be factorized into many sub-characteristic functions as

$$\begin{aligned}&\Xi_B(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}_a}, \underline{\mathbf{c}}_{\mathcal{S},1}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{b}}_{\mathcal{S},1}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}_a,2}, \underline{\mathbf{x}}_{\mathcal{S},1}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}_a,2})\\ =&\Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}_a})\prod_{i=1}^{2}\left(\prod_{a\in\mathcal{T}_i}\Xi_{C_a}(\mathbf{u}_a, \mathbf{c}_{a,i})\prod_{a\in\mathcal{T}_i}\underbrace{\Xi_{\pi_{a,i}}(\mathbf{c}_{a,i}, \mathbf{b}_{a,i})}_{=\mathbb{1}_{\{\mathbf{b}_{a,i}=\pi_{a,i}(\mathbf{c}_{a,i})\}}}\prod_{a\in\mathcal{T}_i}\underbrace{\Xi_{\phi_a}(\mathbf{b}_{a,i}, \mathbf{x}_{a,i})}_{=\mathbb{1}_{\{\mathbf{x}_{a,i}=\phi_a(\mathbf{b}_{a,i})\}}}\right),\end{aligned} \tag{3.26}$$

where $\mathcal{T}_1 = \mathcal{S}$, $\mathcal{T}_2 = \mathcal{S} \cup \mathcal{R}_a$, and where $\mathcal{R}_a \triangleq \{r \in \mathcal{R} : \mathcal{S}_r \neq \emptyset\}$ is the set of active relays in the second time slot. $\Xi_{F_{nc}}$, Ξ_{C_a}, Ξ_{π_a}, and Ξ_{ϕ_a} represent the characteristic functions of the behavioral modeling of the global network encoder, the channel encoder, the channel interleaver, and the modulator of $a \in \{s, r\}$, respectively. The characteristic function $\Xi_{F_{nc}}$ can be factorized as

$$\Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}_a}) = \prod_{r\in\mathcal{R}_a}\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r), \tag{3.27}$$

where $\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r)$ represents the characteristic function of the behavioral modeling of the local network coding function given by (3.24). Finally, the characteristic function Ξ_{F_r} can be factorized as

$$\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r) = \left(\prod_{s\in\mathcal{S}_r}\underbrace{\Xi_{\Pi_{s,r}}(\mathbf{u}_s, \tilde{\mathbf{u}}_{s,r})}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_{s,r}=\Pi_{s,r}(\mathbf{u}_s)\}}}\right)\underbrace{\Xi_{\Pi_r}(\mathbf{u}_r, \tilde{\mathbf{u}}_r)}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_r=\Pi_r(\mathbf{u}_r)\}}}\mathbb{1}_{\{\tilde{\mathbf{u}}_r=\sum_{s\in\mathcal{S}_r}\oplus\mathbf{G}_{s,r}\tilde{\mathbf{u}}_{s,r}\}}. \tag{3.28}$$

Fig. 3.8 depicts the factor graph representing the factorization (3.26), when $\mathcal{R}_a = \mathcal{R}$ and $\mathcal{S}_r = \mathcal{S}, \forall r \in \mathcal{R}$. Fig. 3.9 shows the factor graph representing the factorization (3.27) and (3.28) for a $(2, 2, 1)$-MAMRC. A brute-force approach to evaluate the marginal a posteriori probability $P(u_{s,k}|\mathbf{Y}_d, B)$ in (3.25) is of course intractable. We thus resort to the sum-product algorithm [45]. Since the overall factor graph has cycles, the algorithm is applied *iteratively*. As well known, the message-passing schedule may impact the convergence. Once it is specified, messages along the edges connecting the different nodes of the factor graph circulate until convergence (to approximate marginals). In our case, each iteration comprises the following steps:

1. Detection and Demapping: Compute the messages $\mu_{\phi_a\to b_{a,i,k}}(b_{a,i,k})$, where $a \in \{s, r\}$, $i = 1, 2$, and $k = 1, \ldots, n_a$, using the channel observations $\mathbf{Y}_{d,i}$ and the messages of the variable nodes $\mu_{b_{a',i,k'}\to\phi_{a'}}(b_{a',i,k'})$, $(a', k') \neq (a, k)$, and route them, through the interleaver connections, to the variables nodes $c_{a,i,k}$.

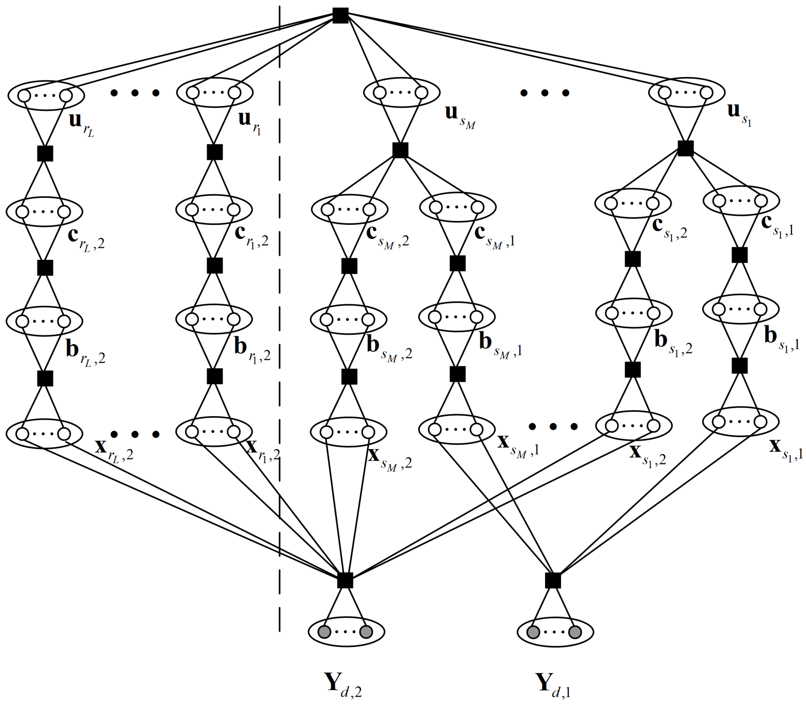


Figure 3.8: Factor graph representing the factorization in (3.25) and (3.26) for a $(M, L, 1)$-NOMAMRC when $\mathcal{R}_a = \mathcal{R}$, and $\mathcal{S}_r = \mathcal{S}, \forall r \in \mathcal{R}$.

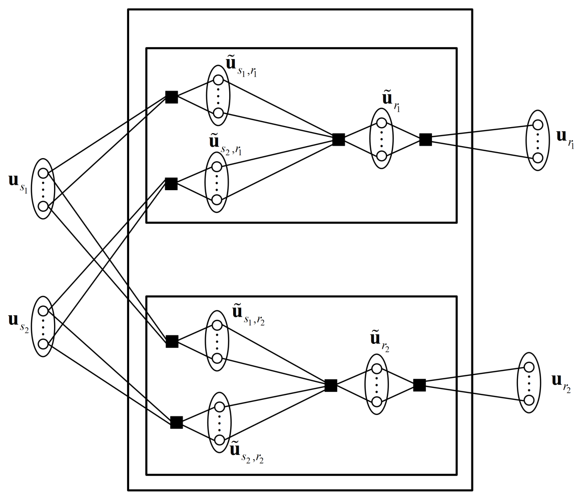


Figure 3.9: Factor graph representing the factorization (3.27) and (3.28) for a $(2, 2, 1)$-NOMAMRC when $\mathcal{R}_a = \mathcal{R}$, and $\mathcal{S}_r = \mathcal{S}, \forall r \in \mathcal{R}$.

2. Channel decoding of the sources: Compute the messages $\mu_{C_s \to c_{s,i,k}}(c_{s,i,k})$ and $\mu_{C_s \to u_{s,\ell}}(u_{s,\ell})$, where $\ell = 1, \ldots, K$.

3. Network decoding: Compute the messages $\mu_{F_{nc} \to u_{r,\ell}}(u_{r,\ell})$.

4. Channel decoding of the relays: Compute the messages $\mu_{C_r \to c_{r,2,k}}(c_{r,2,k})$ and $\mu_{C_r \to u_{r,\ell}}(u_{r,\ell})$.

5. Network decoding: Compute the messages $\mu_{F_{nc} \to u_{s,\ell}}(u_{s,\ell})$.

6. Based on the product of all incoming messages at the nodes $u_{a,\ell}$, where $a \in \{s, r\}$, and $\ell = 1, \ldots, K$, hard decisions are made to obtain the estimates $\hat{\mathbf{u}}_a$. Then, CRC checks are performed to extract the correctly decoded frames. Finally, separate network decoding is performed on the correctly decoded messages. For GFNC, decoding can be performed using Gauss-Jordan elimination. For BI-XOR-based or XOR-based network coding, if a network coded packet of a relay r and $|\mathcal{S}_r| - 1$ packets of the sources in $\mathcal{S}_r$ are correctly decoded then all the packets of $\mathcal{S}_r$ are correctly decoded.

7. If the M source packets are correctly decoded or if the maximum number of iterations is reached, the iterative process stops. Else another iteration is performed.

3.3.5 Network decoding

In this section, we detail the messages generated by the network decoder. The messages generated from F_{nc} to $u_{r,\ell}$, where $r \in \mathcal{R}_a$, and $\ell = 1, \ldots, K$ are given by

$$\mu_{F_{nc} \to u_{r,\ell}}(u_{r,\ell}) = \sum_{\sim\{u_{r,\ell}\}} \Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}_a}) \prod_{(r',\ell') \neq (r,\ell)} \mu_{u_{r',\ell'} \to F_{nc}}(u_{r',\ell'}) \prod_{s \in \mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'} \to F_{nc}}(u_{s,\ell'}). \tag{3.29}$$

We further simplify (3.29) by only considering the check equation provided by r (separately from the other relays). In this case, the messages generated from F_{nc} to $u_{r,\ell}$, are given by

$$\mu_{F_{nc} \to u_{r,\ell}}(u_{r,\ell}) \approx \mu_{F_r \to u_{r,\ell}}(u_{r,\ell}) = \sum_{\sim\{u_{r,\ell}\}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r) \prod_{\ell' \neq \ell} \mu_{u_{r,\ell'} \to F_r}(u_{r,\ell'}) \prod_{s \in \mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'} \to F_r}(u_{s,\ell'}). \tag{3.30}$$

Similarly, the messages generated from the check nodes F_r, to $u_{s,\ell}$, where $s \in \mathcal{S}_r$, and

$\ell = 1, \ldots, K$, are given by

$$\mu_{F_r \to u_{s,\ell}}(u_{s,\ell}) = \sum_{\sim\{u_{s,\ell}\}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r) \prod_{\ell'=1}^{K} \mu_{u_{r,\ell'} \to F_r}(u_{r,\ell'}) \prod_{(s',\ell') \neq (s,\ell)} \mu_{u_{s',\ell'} \to F_r}(u_{s',\ell'}). \quad (3.31)$$

3.3.6 Practical schemes evaluation

In this section, QAM with Gray labeling is used at the sources and the relays. Each packet of the source has a length $k = 256$ information bits. The sources use identical punctured turbo codes, with coding rate $w_s \in \{1/2, 3/4, 5/6\}$, made of two 4-states rate-1/2 RSCs encoders with generator matrix $\mathbf{G}_s(D) = \begin{bmatrix} 1 & \frac{1+D^3}{1+D+D^2+D^3} \end{bmatrix}$. The relays use a punctured convolutional code, with coding rate $w_r = 1$ made of a 16-states rate-1/2 RSC encoder with generator matrix $\mathbf{G}_r(D) = \begin{bmatrix} 1 & \frac{1+D^3+D^4}{1+D+D^2+D^3+D^4} \end{bmatrix}$. The systematic bits are punctured completely. The proposed constituent codes are given only as an example to illustrate the benefits of our approach.

For SNCC/SNCD, the linear network coding coefficients, that are used in (3.21), are chosen such that if the number of unknown variables (messages) in the received equations is less or equal to the number of the received equations then these equations are solvable, the existence of such network codes is guaranteed by [57, Theorem 11]. It is worth mentioning that we do not use specific network coding coefficients and SNCD at the receiving node relies on the previous property to determine if the sources' messages which are not correctly decoded form the direct transmission and are included in the relays network coded messages are correctly decoded or not (Genni aided approach).

3.3.6.1 Orthogonal MAMRC

In the first simulations, we consider $(2, 2, 1)$-OMAMRC with imperfect S-R links. For $m_d = 1$, we choose $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$. For $m_d = 4$, we take $\gamma_{r,d} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$, and $\gamma_{s,r} = 100\gamma$. Fig. 3.10 shows the results. We observe that

- the GFNC achieves the promised full diversity,
- the BI-XOR has a very close performance to GFNC and starts to deviate at high SNR,
- as expected, the XOR network coding, where the relays directly perform XOR operations on the bits of the correctly decoded frames of the sources without interleaving,

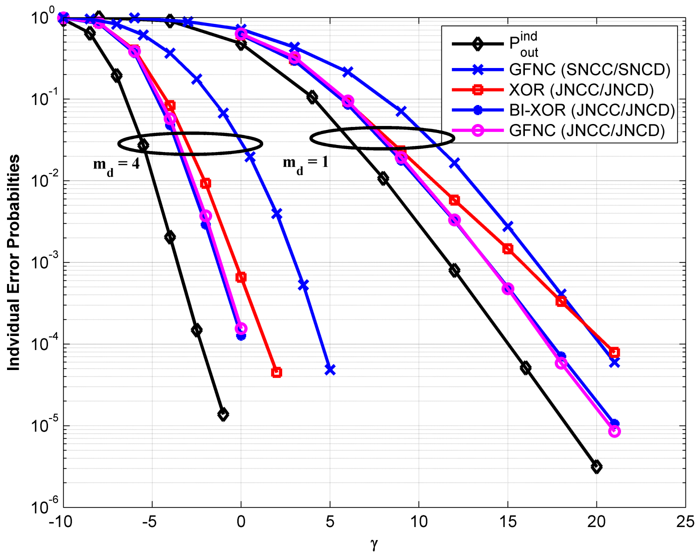


Figure 3.10: IBLER vs individual outage probability for (2,2,1)-OMAMRC, $R = 1/3$ (b./c.u), $\alpha = 2/3$.

does not achieve the full diversity,

- GFNC in JNCC/JNCD framework has a coding gain of 3 dB in the case of $m_d = 1$ and 4 dB in the case of $m_d = 4$ with respect to the GFNC in SNCC/SNCD framework.

similar observations were made for higher modulation, up to 64-QAM, and higher coding rates, up to 5/6 at the sources. Fig. 3.11, shows the results for the case of perfect S-R, 64-QAM modulation, and coding rate $w_s = 5/6$.

In the second simulations, we want to further investigate the performance of BI-XOR. We chose $(2, 3, 1)$-OMAMRC with $\gamma_{s,r} = \gamma_{s,d} = \gamma_{r,d} = \gamma$, $m_d = 1$, and $m_d = 2$. Fig. 3.12. shows the IBLER at each relay and the destination when (1) $\gamma_{r,\tilde{r}} = 10\gamma$, and (2) $\gamma_{r,\tilde{r}} = 0$. We observe that

- when BI-XOR is used the diversity is increased with the index of the relays, since relays with high index can listen to relays with lower index, and the destination has the highest diversity, since it can listen to all the relays. Note that if XOR is used at the relays then, the diversity order can not increase with the number of relays.

- the quality of R-R links increases the coding gain at the destination but not the diversity order.

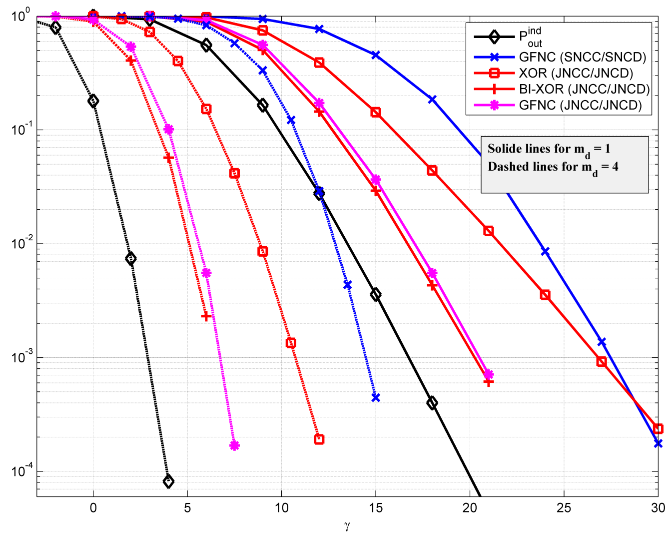


Figure 3.11: IBLER vs individual outage probability for (2,2,1)-OMAMRC, $R = 15/11$ (b./c.u), $\alpha = 6/11$, $q_s = q_r = 6$, and $w_s = 5/6$.

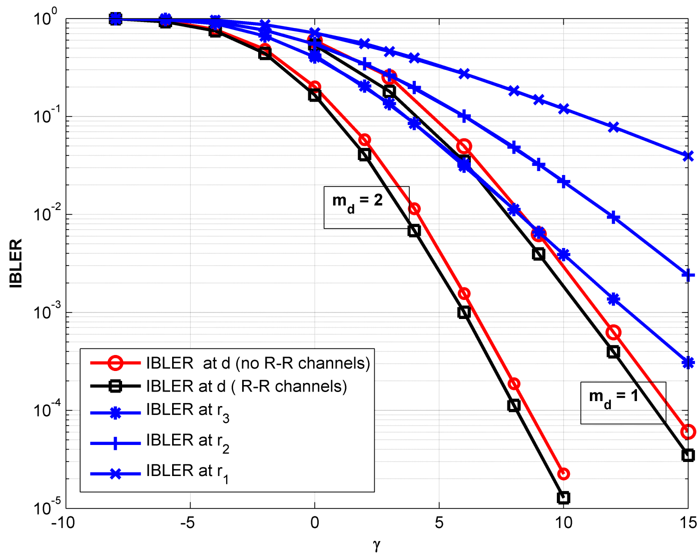


Figure 3.12: IBLER at the destination and the relays for $(2,3,1)$-OMAMRC, $R = 2/7$ (b./c.u) , $\alpha = 4/7$.

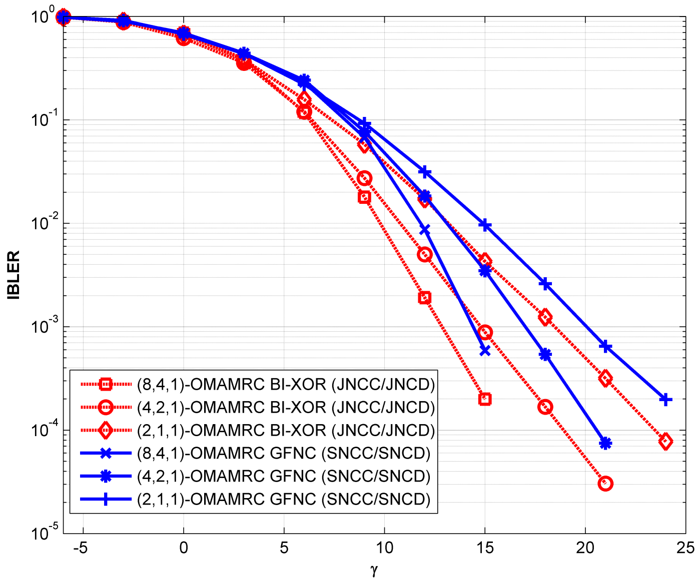


Figure 3.13: IBLER of BI-XOR for different OMAMRCs, where $MR = 0.8$ (b./c.u), $\alpha = 0.8$, compared to IBLER of GFNC in SNCC/SNCD framework.

In the third simulations, we benefit from the flexibility and simplicity of the BI-XOR to compare the IBLER of three OMAMRCs with the same sum spectral efficiency MR, namely $(2,1,1)$-OMAMRC, $(4,2,1)$-OMAMRC, and $(8,4,1)$-OMAMRC. As a benchmark, we calculate the IBLER of GFNC when SNCC/SNCD framework is used. Although the three networks have the same spectral efficiency, it is clear that the last network has the highest possible diversity. We chose $m_d = 1$, and $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$. Fig. 3.13 shows the results. Again we see that the diversity order of BI-XOR increases with the number of the relays. BI-XOR does not achieve the full diversity, except for $(2,1,1)$-OMAMRC. Nevertheless BI-XOR has a better performance than the GFNC with SNCC/SNCD at low to moderate SNR. Furthermore, from the result of the first simulation, we could conjecture that in this rage of SNR the GFNC with JNCC/JNCD (which is very complex to implement for $(8,4,1)$-MAMRC) will have a very close performance to BI-XOR.

3.3.6.2 MAMRC

In the first set of simulations, we consider $(2,2,1)$-SOMAMRC-I and $(2,2,1)$-NOMAMRC. We choose $\gamma_{r,d} = \gamma_{s,d} = \gamma_{s,r} = \gamma$, when $m_d = 1$, and $\gamma_{r,d} = \gamma_{s,d} = \gamma$, and $\gamma_{s,r} = 100\gamma$, when $m_d = 4$. Fig. 3.14 and Fig. 3.15 show the results. We observe that:

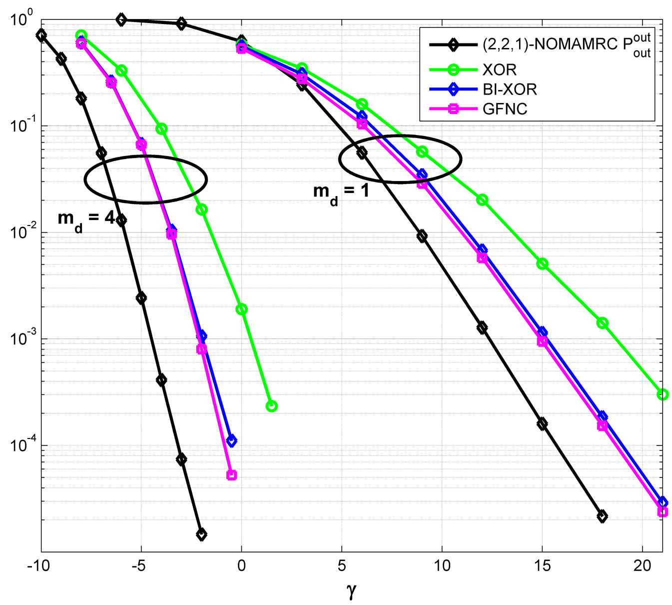


Figure 3.14: IBLER and p_{out}^{ind}, $(2, 2, 1)$-NOMAMRC, $R = 2/3$ (b./c.u), $\alpha = 2/3$.

- The performance of BI-XOR based network coding and GFNC are identical for $(2, 2, 1)$-SOMAMRC-I and very close for $(2, 2, 1)$-NOMAMRC. Unlike GFNC, BI-XOR based network coding does not achieve the full diversity order. The performance curve starts slightly deviating from the one of GFNC at high SNR;

- As expected, XOR based network coding (where the relays directly perform XOR operations on the bits of the correctly decoded sources' packets without interleaving) does not achieve the full diversity;

- The IBLER curves of BI-XOR based network coding and GFNC are 2.5 dB away from the individual outage probability curves for both $m_d = 1$ and $m_d = 4$.

In the second set of simulations, we further investigate the potential of BI-XOR based network coding, much more flexible and simpler to implement than GFNC. We compare the IBLER of four $(2, L, 1)$-SOMAMRC Type I, where $L = 1, 2, 3, 4$, and three $(2, L, 1)$-NOMAMRC, where $L = 1, 2, 3$. Although the different networks have the same spectral efficiency, it is clear that the ones with the highest number of relays achieve better diversity orders. We choose $\gamma_{r,d} = \gamma_{s,d} = \gamma_{s,r} = \gamma$ and $m_d = 2$. Fig. 3.16 and Fig. 3.17 show the results. We see that the diversity order achieved by BI-XOR based network coding increases with the number of the relays. Even if BI-XOR based network coding does not

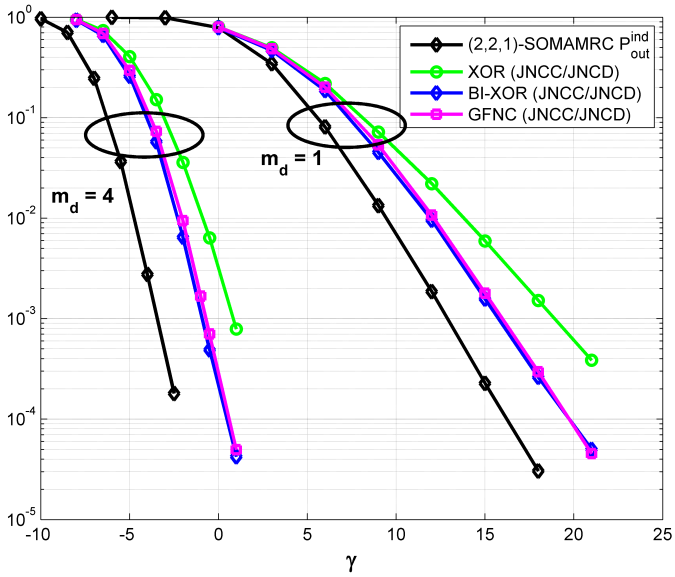


Figure 3.15: IBLER and p_{out}^{ind}, $(2, 2, 1)$-SOMAMRC-I, $R = 2/3$ (b./c.u), $\alpha = 2/3$.

achieve the full diversity order for $L \geq 2$, its performance is still approximately 2 dB away from the individual outage probability in the SNR region of interest. Based on the first set of simulations, we conjecture that, in this SNR range, the performance of GFNC will be very close to the one of BI-XOR network coding. On the other hand, we are sure that XOR based network coding will not give better results, since it is well known that the diversity order achieved by XOR based network coding cannot increase with the number of relays.

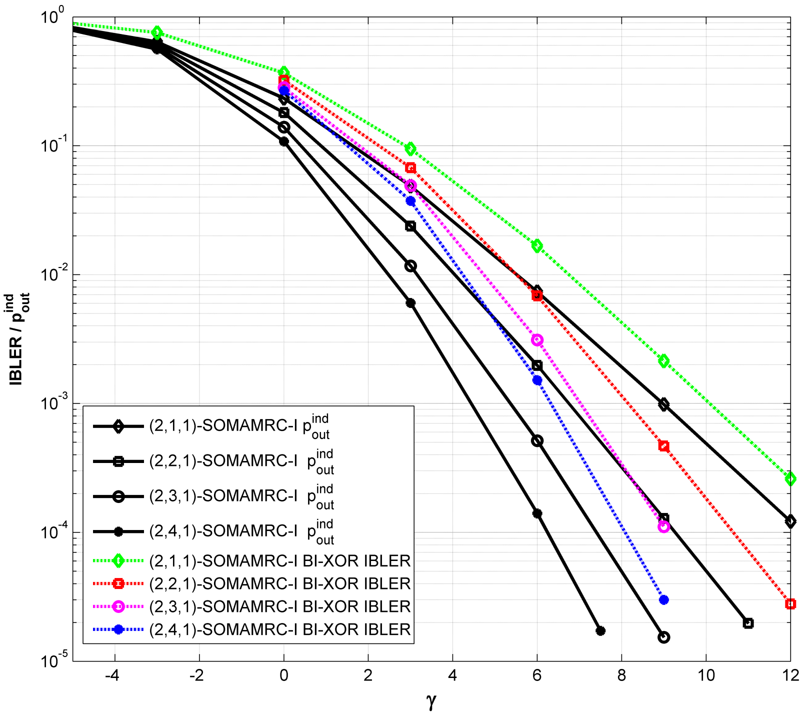


Figure 3.16: IBLER and p_{out}^{ind}, $(2, L, 1)$-SOMAMRC-I, where L $= 1, 2, 3, 4$, $R = 2/3$ (b./c.u), $\alpha = 2/3$ and $m_d = 2$.

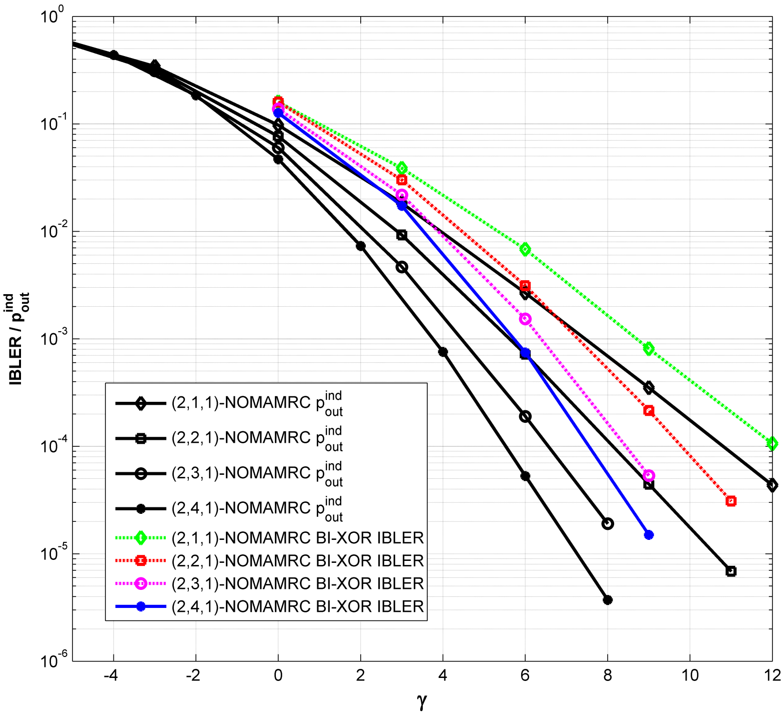


Figure 3.17: IBLER and p_{out}^{ind}, $(2, L, 1)$-NOMAMRC, where L $= 1, 2, 3$, $R = 2/3$ (b./c.u), $\alpha = 2/3$ and $m_d = 2$.

CHAPTER 4

Dynamic Selective Relaying for Slow-Fading Multiple-Access Multiple-Relay Channel

In this chapter, we propose and investigate a new relaying protocol coined Dynamic Selective Decode-and-Forward (D-SDF) for the half-duplex slow fading NOMAMRC. In D-SDF, each relay is provided by a selection procedure that decides when it should stop listening and start cooperating. Based on the definition of NOMAMRC (see sec. 3.1.1), an information outage analysis in JNCC/JNCD framework is conducted conditional on the adopted selection procedure. Then, practical JNC coding schemes, at the relays, are proposed together with JNC decoding receiver architectures, at the relays and the destination. The receivers of the relays and the destination are designed to benefit from the signals of the previously activated relays to better decode the sources. Simulations show the effectiveness of the proposed schemes.

4.1 System model

4.1.1 Description of the system

Fig. 3.1 shows an $(M, L, 1)$-MAMRC. Let $\mathcal{S} = \{s_1, \ldots, s_M\}$ denote the set of statistically independent sources, $\mathcal{R} = \{r_1, \ldots, r_L\}$ the set of of half-duplex dedicated relays, and d the common destination. Each source $s \in \mathcal{S}$ wants to communicate its packet $\mathbf{u}_s \in \mathbb{F}_2^K$ of K information bits to d with the help of the L relays. The sources are equipped with one transmit antenna, the common destination with m_d receive antennas, and the relays with one transmit antenna and m_r receive antennas. All sources are symmetric, i.e., have the same transmission rate, power, and priority. The relays are half-duplex, i.e., at any point in time, a relay can either transmit or receive (but not both). The maximum number of

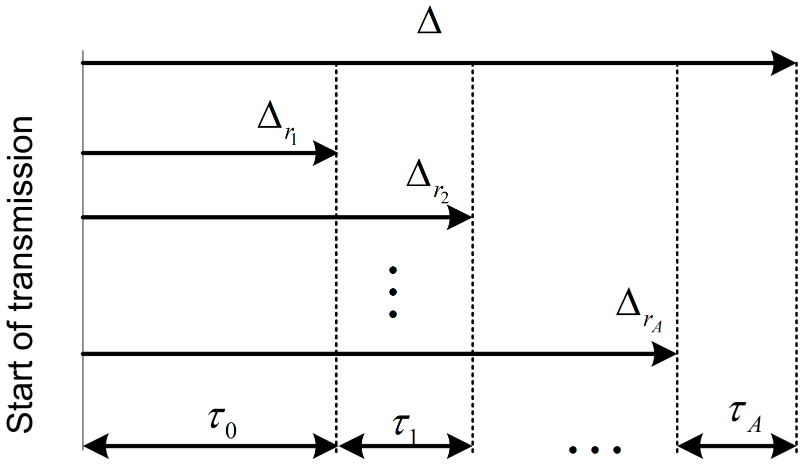


Figure 4.1: A possible realization of the listening periods of the relays.

channel uses (complex dimensions) during which the sources can transmit is denoted by Δ. The sources are unaware of the existence of the relays. The codeword $\mathbf{x}_s \in \mathbb{C}^{\Delta}$ for the packet for $\mathbf{u}_s$ of $s \in \mathcal{S}$ is *universal* in the sense that any listening node can decode it as soon as the transmission rate is less or equal the accumulated mutual information. This property can be implemented in practice with incremental redundancy codes, such as rateless codes [129–131] or families of rate compatible punctured codes [132–136]. After a listening period of Δ_r channel uses which depends on the considered selection strategy, a relay $r \in \mathcal{R}$ can become active and forward a function of the set $\mathcal{S}_r \subseteq \mathcal{S}$ of the sources' packets it has correctly decoded. We label the relays according to their transmission order, i.e., if $\Delta_r \leq \Delta_{\tilde{r}}$, then $ind(r) \leq ind(\tilde{r})$. If $\Delta_r > \Delta$ then r can not cooperate and will remain silent. The actual number of cooperating relays during the period Δ is denoted by A. Clearly, $A \leq L$. The period during which exactly i relays are cooperating is denoted by τ_i (see Fig. 4.1). The set of cooperating relays during the period τ_i is denoted by $\mathcal{R}_i$, with the convention that $\mathcal{R}_0 = \emptyset$. We have $\mathcal{R}_i = \{r_1, \ldots, r_i\} \subseteq \mathcal{R}$, $\forall i = 1, \ldots, A$. Each relay employs JNCC, using an independent codebook to jointly encode the packets $\underline{\mathbf{u}}_{\mathcal{S}_r}$, and generate its own codeword denoted by $\mathbf{x}_r \in \mathbb{C}^{(\Delta - \Delta_r)}$. The sequence $[\underline{\mathbf{x}}_{\mathcal{S}_r}^{\top}, \mathbf{x}_r^{\top}]^{\top}$ is again a *universal* codeword for the packets $\underline{\mathbf{u}}_{\mathcal{S}_r}$. JNCC processing time is neglected. The set of transmitting nodes during period τ_i is denoted by $\mathcal{T}_i = \mathcal{S} \cup \mathcal{R}_i$, $\forall i = 0, \ldots, A$. Let $\mathbf{x}_a^{(i)} \in \mathbb{C}^{\tau_i}$, where $a \in \mathcal{T}_i$, denotes the transmitted sub-sequence of node a during τ_i channel uses. During the period τ_i, $i = 0, \ldots, A$, the received signal at node $b \in \mathcal{R} \cup \{d\} \backslash \mathcal{R}_i$ is given by

$$\mathbf{y}_{b,k}^{(i)} = \sum_{a \in \mathcal{T}_i} \sqrt{\gamma_{a,b}} \mathbf{h}_{a,b} x_{a,k}^{(i)} + \mathbf{n}_{b,k}^{(i)}, \forall k = 1, \ldots, \tau_i \quad (4.1)$$

where $x_{a,k}^{(i)}$ belongs to the signal set $\mathcal{X}_a$, $\gamma_{a,b}$ is the average Signal-to-Noise Ratio (SNR) for the link (a,b), taking into account path loss and shadowing, $\mathbf{h}_{a,b} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$ is the channel gain vector of the link (a,b) due to nonselective multipath fading (that stays constant during the whole transmission, but changes independently from one transmission to the next), and $\mathbf{n}_{b,k}^{(i)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$ is the additive white Gaussian noise (AWGN) vector. The channel gain vectors are assumed mutually independent. In the sequel, $\boldsymbol{\gamma}$ and $\mathbf{h}$ respectively denote the vector of average SNRs and channel gains for all possible pairs (a,b). $\mathbf{Y}_b^{(i)}$ denotes the collection of received samples at b during τ_i channel uses, $\mathbf{Y}_d$ the collection of received samples at d during Δ channel uses, and $\mathbf{Y}_r$ the collection of received samples at r during Δ_r channel uses. To let the receivers (destination, relays) know which relay is there, cooperating, and which sources' packets are included in the relay's transmitted signal, a side information of M bits is required. All receivers implement JNCD, assuming perfect channel (and side) information.

4.1.2 Examples of selection strategies in D-SDF

4.1.2.1 Waiting for all

Each relay decides to switch from listening to forwarding as soon as it successfully decodes *all* the sources.

4.1.2.2 Waiting for at least one

Each relay decides to switch from listening to forwarding as soon as it successfully decodes *at least* one source.

4.1.2.3 Waiting for at least a specific subset

Each relay decides to switch from listening to forwarding as soon as it successfully decodes at least a predefined subset $\mathcal{S}_{r,c} \subseteq \mathcal{S}$ of sources. The subset $\mathcal{S}_{r,c}$ is assigned prior to the transmission.

4.1.2.4 Threshold selection

Each relay r works in *waiting for all* mode if the time index is less or equal a threshold, denoted by $\Delta_{t,r}$, where $\Delta_{th,r} \in]0, \Delta[$, and in *waiting for at least one* mode if the time index is greater than $\Delta_{th,r}$. The variables $\Delta_{th,r}$, $\forall r \in \mathcal{R}$, are system parameters which could be fixed or optimized values.

4.2 Outage analysis

In this Section, we assume that $\Delta \to \infty$, and $\Delta_{r_i} \to \infty$, $\forall i = 1, \ldots, A$, and that all transmitted sequences are made of independent identically distributed symbols. We also suppose that the destination d knows A, and Δ_r and $\mathcal{S}_r$, $\forall r \in \mathcal{R}_A$. Our objective is to characterize the individual and common outage events at the destination conditional on the decision taken by the relays and the channel states $\mathbf{h}_{a,d}$, $\forall a \in \mathcal{S} \cup \mathcal{R}_A$.

Let us define the independent input random variables $x_a \sim p(x_a)$, $\forall a \in \mathcal{S} \cup \mathcal{R}$, and the associated independent output random vectors $\mathbf{y}_b$ $\forall b \in \mathcal{R} \cup \{d\}$. where the relations between $\mathbf{y}_b$ and x_a at each channel use are defined in (4.1).

For the sake of notation simplicity, we omit the conditioning by the channel state in the outage event definitions and mutual information expressions. The individual outage event of source s at receiver b is denoted by $\mathtt{O}_{b,s}$, while the common outage event at receiver b is denoted by $\mathtt{E}_b$.

4.2.1 Outage events at the destination

Under JNCD, any set of cooperating relays during the period τ_i, $\mathcal{R}_i \subseteq \mathcal{R}_A$, where $i = 0, 1, \ldots, A$, can be partition into three sets (i) if $r \in \mathcal{R}_i^I \subseteq \mathcal{R}_i$ the relay signal $\mathbf{x}_r$ is interference, (ii) if $r \in \mathcal{R}_i^K \subseteq \mathcal{R}_i$ the relay signal $\mathbf{x}_r$ is perfectly known, (iii) if $r \in \mathcal{R}_i^U \subseteq \mathcal{R}_i$ the relay signal $\mathbf{x}_r$ can be jointly decoded with the signals of a subset of sources $\mathcal{U} \subseteq \mathcal{S}$. The definitions of the sets $\mathcal{R}_i^I$, $\mathcal{R}_i^K$, and $\mathcal{R}_i^U$ (same as in [94]) are given below, where $\mathcal{I} \subset \mathcal{S}$ is the set of sources which are interference (i.e., cannot be decoded correctly), $\mathcal{U}_c = \mathcal{I}^c \setminus \mathcal{U}$ is the set of sources that are perfectly decoded, and $\mathcal{U}$ is the set of sources that need to be decoded.

Definition (12) $\mathcal{R}_i^I = \{r \in \mathcal{R}_i : \mathcal{I} \bigcap \mathcal{S}_r \neq \emptyset\}$ *the set of relays whose signals are* *interference during the period* τ_i.

Definition (13) $\mathcal{R}_i^K = \{r \in \mathcal{R}_i : \mathcal{S}_r \subseteq \mathcal{U}_c\}$ *the set of relays whose signals are perfectly known during the period* τ_i.

Definition (14) $\mathcal{R}_i^U = \mathcal{R}_i \backslash \{\mathcal{R}_i^K \cup \mathcal{R}_i^I\}$ *the set of relays whose signals are to be jointly decoded with the sources belonging to* $\mathcal{U}$ *during the period* τ_i.

Based on the previous definitions, the following result holds:

Proposition (5) *The maximum possible rate which allows the destination* d *to decode all the packets of the sources in* $\mathcal{I}^c$ *(common) knowing that the signals in* $\mathcal{I} \subset \mathcal{S}$ *are interference is given by*

$$R_{\mathcal{I}^c}^d = \min_{\mathcal{U} \subseteq \mathcal{I}^c} \frac{\sum_{j=0}^{L} (\alpha_{j+1}^d - \alpha_j^d) I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}_j^U}; \mathbf{y}_d^{(i)} | \mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}_j^K})}{|\mathcal{U}|}, \tag{4.2}$$

where $\alpha_0^d = 0$, $\alpha_i^d = \min\{\frac{\Delta_{r_i}}{\Delta}, 1\}$, $\forall i = 1, \ldots, L$, *and* $\alpha_{L+1}^d = 1$ *are the time sharing factors.*

Proof. See appendix B. □

From Proposition 5, the maximum rate which allows the destination d to decode the packet of a given source s is given by (see [94, proposition 1])

$$R_s^d = \max_{\mathcal{I} \subset \mathcal{S} : s \in \mathcal{I}^c} R_{\mathcal{I}^c}^d. \tag{4.3}$$

The maximum rate which allows the destination d to decode at least one source is given by

$$R_{(.)}^d = \max_{s \in \mathcal{S}} R_s^d. \tag{4.4}$$

Example (3) *Let us consider an* $(2, 2, 1)$*-MAMRC. For a given channel outcome, consider that* $A = 2$, $\mathcal{S}_{r_1} = \{s_1\}$, $\mathcal{S}_{r_2} = \{s_2\}$, *and* $\Delta_{r_1} = \Delta_{r_2} = \Delta/3$, *i.e., the two relays start cooperating simultaneously, hence,* $\tau_0 = \Delta/3$, $\tau_1 = 0$, *and* $\tau_2 = 2\Delta/3$. *In this case, the set* *of conditions of* (B.3) *becomes*

$$
\begin{aligned}
R &\leq \underbrace{\frac{1}{3}I(x_{s_1};\mathbf{y}_d|x_{s_2}) + \frac{2}{3}I(x_{s_1},x_{r_1};\mathbf{y}_d|x_{s_2},x_{r_2})}_{c_1} \\
R &\leq \underbrace{\frac{1}{3}I(x_{s_2};\mathbf{y}_d|x_{s_1}) + \frac{2}{3}I(x_{s_2},x_{r_2};\mathbf{y}_d|x_{s_1},x_{r_1})}_{c_2} \\
2R &\leq \frac{1}{3}I(x_{s_1},x_{s_2};\mathbf{y}_d) + \frac{2}{3}I(x_{s_1},x_{r_1},x_{s_2},x_{r_2};\mathbf{y}_d) \\
&= c_2 + \underbrace{I(x_{s_1};\mathbf{y}_d) + \frac{2}{3}I(x_{s_1},x_{r_1};\mathbf{y}_d)}_{c_1'} \\
&= c_1 + \underbrace{I(x_{s_2};\mathbf{y}_d) + \frac{2}{3}I(x_{s_2},x_{r_2};\mathbf{y}_d)}_{c_2'}
\end{aligned}
$$

Fig. 4.2 depicts an example of instantaneous rate region corresponding to the previous conditions, where

$$R_{\mathcal{S}}^d = \min\{c_1, c_2, \frac{c_1 + c_1'}{2}\} = c_2,$$

$$R_{s_1}^d = \max\{R_{\mathcal{S}}^d, c_1'\} = c_1',$$

$$R_{s_2}^d = \max\{R_{\mathcal{S}}^d, c_2'\} = R_{\mathcal{S}}^d,$$

and

$$R_{(.)}^d = \max\{R_{s_1}^d, R_{s_2}^d\} = R_{s_1}^d.$$

■

For independent identically Gaussian distributed inputs, i.e., $p(x_a) \sim \mathcal{CN}(0,1)$, the instantaneous mutual information expressions used in the previous outage events are given by

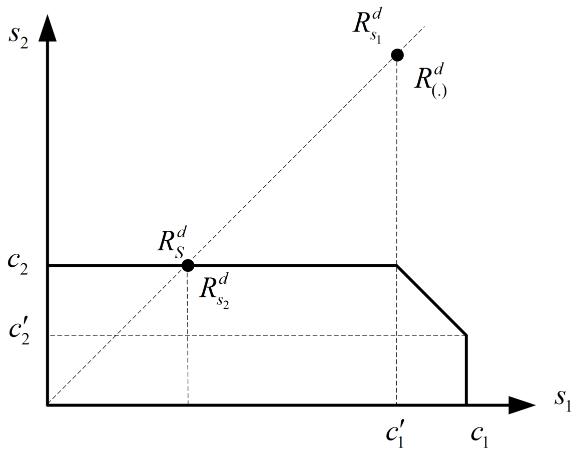


Figure 4.2: An example of instantaneous rate region for an (2, 2, 1)-MAMRC with D-SDF.

$$I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}_j^U}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}_j^K}) = \log\left(1 + \frac{\sum\limits_{a \in \mathcal{U} \cup \mathcal{R}_j^U} \gamma_{a,d} ||\mathbf{h}_{a,d}||^2}{1 + \sum\limits_{a \in \mathcal{I} \cup \mathcal{R}_j^I} \gamma_{a,d} ||\mathbf{h}_{a,d}||^2}\right) \tag{4.5}$$

Finally, the individual outage event of source s at destination d is defined as $\mathtt{O}_{d,s} = \{R > R^d_s\}$, while the common outage event at destination d is defined as $\mathtt{E}_{d,\mathcal{S}} = \{R > R^d_{\mathcal{S}}\}$.

It follows that the individual outage probability of a particular source s at d and the common outage probability at d are defined as

$$P^{ind}_{out,d,s}(R, \boldsymbol{\gamma}) = \Pr(\mathtt{O}_{d,s}) = \mathbb{E}\left[\mathbb{1}_{\{\mathtt{O}_{d,s}\}}\right] = \int\limits_{\mathbf{h}} \mathbb{1}_{\{\mathtt{O}_{d,s}(\mathbf{h})\}} p(\mathbf{h}) d(\mathbf{h}), \tag{4.6}$$

and

$$P^{com}_{out,d}(R, \boldsymbol{\gamma}) = \Pr(\mathtt{E}_d) = \mathbb{E}\left[\mathbb{1}_{\{\mathtt{E}_d\}}\right] = \int\limits_{\mathbf{h}} \mathbb{1}_{\{\mathtt{E}_d(\mathbf{h})\}} p(\mathbf{h}) d(\mathbf{h}). \tag{4.7}$$

respectively.

From (4.6) and (4.7), we define the individual and common ϵ-outage achievable rates as

$$R^{ind}_{\epsilon}(\boldsymbol{\gamma}) = sup\{R : P^{ind}_{out,d,s}(R, \boldsymbol{\gamma}) \leq \epsilon, \quad \forall s \in \mathcal{S}\} \tag{4.8}$$

and

$$R_{\epsilon}^{com}(\boldsymbol{\gamma}) = sup\{R : P_{out,d}^{com}(R, \boldsymbol{\gamma}) \leq \epsilon\}, \tag{4.9}$$

respectively.

4.2.2 Waiting for all D-SDF

Each relay r switches from listening to forwarding when it has correctly decoded all the packets of the sources. In this case, the following result holds:

Proposition (6) *The minimum listening period of the i^{th} cooperating relay r_i in waiting for all D-SDF, when JNCC/JNCD is used, is*

$$\Delta_{r_i} = \min_{r \in \mathcal{R}_{i-1}^c} \frac{K}{R_{\mathcal{S},\mathcal{R}_{i-1}}^r}, \tag{4.10}$$

where $\mathcal{R}_{i-1}^c = \mathcal{R} \backslash \mathcal{R}_{i-1}$, and $R_{\mathcal{S},\mathcal{R}_{i-1}}^r$ is the common symmetric rate at which a relay r can decode all the sources' packets with the help of the set $\mathcal{R}_{i-1}$ of previously activated relays, given by

$$R_{\mathcal{S},\mathcal{R}_{i-1}}^r = \min_{\mathcal{U} \subseteq \mathcal{S}} \frac{I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_r | \mathbf{x}_{\mathcal{U}^c}) + \sum_{j=1}^{i-1} \bar{\alpha}_{r_j}^r I(x_{r_j}; \mathbf{y}_r | \mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{R}_{j-1}})}{|\mathcal{U}|}, \tag{4.11}$$

where $\alpha_{r_j}^r = \frac{\Delta_{r_j}}{\Delta_r}$ and $\bar{\alpha}_{r_j}^r = 1 - \alpha_{r_j}^r$.

Proof. See appendix B. □

From Proposition 6, based on the quality of R-R links, we can directly obtain a lower and upper bounds on the performance of *waiting for all* D-SDF

4.2.2.1 Lower Bound

If the relays are separated and cannot interfere among themself, there is no R-R links, then we have

$$\Delta_{r_i} = \min_{r \in \mathcal{R}_{i-1}^c} \frac{K}{R_{\mathcal{S}}^r}, \tag{4.12}$$

where $R^r_{\mathcal{S}}$ is the maximum symmetric rate which enables r to decode all the packets of $\mathcal{S}$ ($|\mathcal{S}|$-sender MAC), given by

$$R^r_{\mathcal{S}} = \min_{\mathcal{U}\in\mathcal{S}} \frac{I(\mathbf{x}_{\mathcal{U}}, \mathbf{y}_r|\mathbf{x}_{\mathcal{U}_c})}{|\mathcal{U}|} \tag{4.13}$$

4.2.2.2 Upper Bound

If the relays have perfect R-R links, then all the relays will have the same listening period, given by

$$\Delta_r = \min_{r'\in\mathcal{R}} \frac{K}{R^{r'}_{\mathcal{S}}} \quad \forall r \in \mathcal{R}. \tag{4.14}$$

4.2.3 Waiting for at least one D-SDF

Each relay switches from listening to forwarding as soon as it successfully decodes at least one source. In this case, the following result holds:

Proposition (7) *The minimum listening period of the i^{th} cooperating relay r_i in waiting for at least one D-SDF, when JNCC/JNCD is used, is*

$$\Delta_{r_i} = \min_{r\in\mathcal{R}^c_{i-1}} \frac{K}{R^r_{(.),\mathcal{R}_{i-1}}}, \tag{4.15}$$

where $R^r_{(.),\mathcal{R}_{i-1}}$ is the maximum symmetric rate which enables the relay r to decode at least one source with the help of the signals in $\mathcal{R}_{i-1}$, given by

$$R^r_{(.),\mathcal{R}_{i-1}} = \max_{s\in\mathcal{S}} R^r_{(s),\mathcal{R}_{i-1}}, \tag{4.16}$$

where $R^r_{(s),\mathcal{R}_{i-1}}$ is the maximum symmetric rate witch enables r to decode s, given by

$$R^r_{(s),\mathcal{R}_{i-1}} = \max_{\mathcal{I}\subset\mathcal{S}:s\in\mathcal{I}^c} \min_{\mathcal{U}\subseteq\mathcal{I}^c:s\in\mathcal{U}} \frac{\sum_{j=0}^{i}(\alpha^r_{j+1}-\alpha^r_j) I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}^{\mathcal{U}}_j}; \mathbf{y}_r|\mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}^{K}_j})}{|\mathcal{U}|} \tag{4.17}$$

where $\alpha^r_0 = 0$, $\alpha^r_j = \min\{\frac{\Delta_{r_j}}{\Delta_r}, 1\}$, $\forall j = 1, \ldots, L$, and $\alpha^r_{L+1} = 1$ are the time sharing factors.

Proof. Similar to Proposition 6 and omitted due to lack of space. □

4.2.4 Waiting for at least a specific subset D-SDF

Each relay decides to switch from listening to forwarding as soon as it successfully decodes a predefined subset $\mathcal{S}_{r,c}$ of sources. In this case, the following result holds:

Proposition (8) *The minimum listening period of the i^{th} cooperating relay r_i in waiting for at least a specific subset D-SDF, when JNCC/JNCD is used, is*

$$\Delta_{r_i} = \min_{r \in \mathcal{R}^c_{i-1}} \max_{s \in \mathcal{S}_{r,c}} \frac{K}{R^r_{(s),\mathcal{R}_{i-1}}}, \tag{4.18}$$

where $R^r_{(s),\mathcal{R}_{i-1}}$ is given by (4.17).

Proof. Similar to Proposition 6 and omitted due to lack of space. □

4.3 Numerical results

In this Section, we consider only Gaussian i.i.d inputs, where the instantaneous mutual information are computed using (4.5). There are, of course, an infinity of MAMRC configurations (SNR distribution, transmission rate, number of sources, number of relays, etc.). By carefully selecting a few configurations as examples, we intend to shed a light on the performance of D-SDF. We compare D-SDF with static SDF relaying (where all the relays has a fixed listening period, see [94]), and MAC (where all the relays are turned off).

In the first set of simulations, we focus on the symmetric individual ϵ-outage achievable rate where $\epsilon = 0.01$. We consider an $(M, 3, 1)$-MAMRC, where $M = 1, 2, 3, 4$, $m_d = m_r = 1$, and $\gamma_{r,d} = \gamma_{s,d} = \gamma$. In static SDF, we set the listening factor α at $2/3$. First, we aim at evaluating the advantage of D-SDF over static SDF regardless of the selection strategy. Hence, we consider perfect S-R links (there is no R-R links since all the relays will be activated at the same time). Each relay cooperates with all the sources. The results are shown in Fig. 4.3. As expected, D-SDF brings significant gains over static SDF since, in this particular simulation scenario: Indeed, the relays can start cooperating immediately with all the sources without any delay, instead of being obliged to wait $\alpha\Delta$ channel uses.

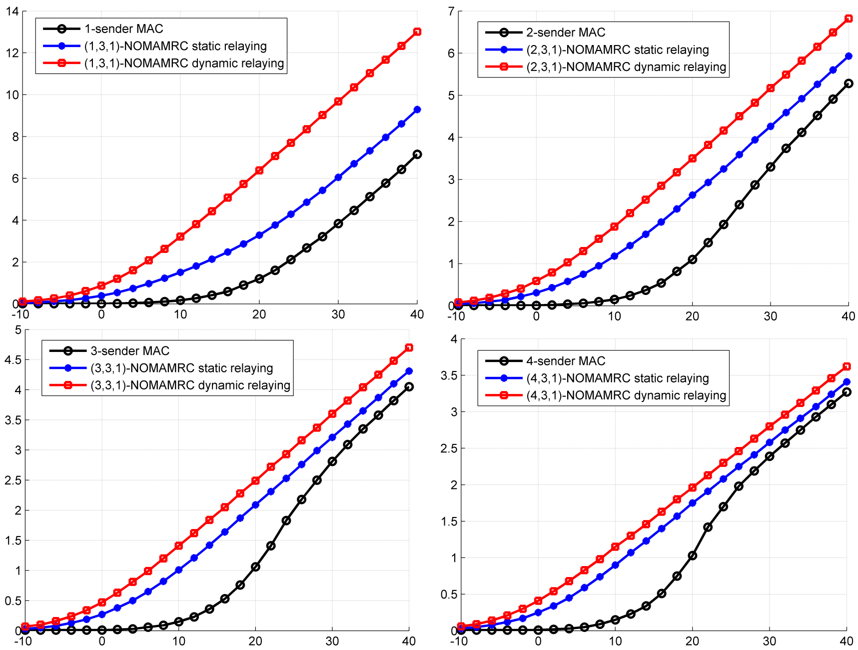


Figure 4.3: $R_{\epsilon}^{ind}(\boldsymbol{\gamma})$ of D-SDF vs static SDF in $(M, 3, 1)$-MAMRC, where $M = 1, 2, 3, 4$, $\gamma_{r,d} = \gamma_{s,d} = \gamma$, and $\gamma_{s,d} = \infty$.

Next, we study the effect of imperfect S-R links. We choose $\gamma_{s,r} = \gamma$. In D-SDF, two extreme cases are considered for the quality of R-R links: (1) $\gamma_{r,\tilde{r}} = 0$; and (2) $\gamma_{r,\tilde{r}} = \infty$, and we only investigate the *waiting for all* selection strategy. The results are shown in Fig. 4.4. We observe that:

- *Waiting for all* D-SDF achieves better rates than static SDF;
- In *waiting for all* D-SDF, the difference between the achievable rates obtained with perfect and imperfect R-R links is very small. This is justified by the fact that, in this simulation scenario, the relays have the same decoding capabilities (on average) because the S-R links are all symmetric.

In the second set of simulations, we want to compare different selection strategies for D-SDF in $(2, 2, 1)$-MAMRC. For that purpose, we evaluate their individual outage probability P_{out}^{ind}. We choose $R = 1$ bit/channel use (b.c.u), $m_d = m_r = 1$, $\gamma_{r,d} = \gamma_{s,d} = \gamma$, $\gamma_{s,r} = 2\gamma$, and $\gamma_{r,\tilde{r}} = 0$. In *Waiting for at least a specific subset* D-SDF, we choose $\mathcal{S}_{r_i,c} = \{s_i\}$, where $i = 1, 2$. The results are shown in Fig. 4.5. We observe that:

- At low SNR, the *waiting for at least one* D-SDF and *Waiting for at least a specific sub-*

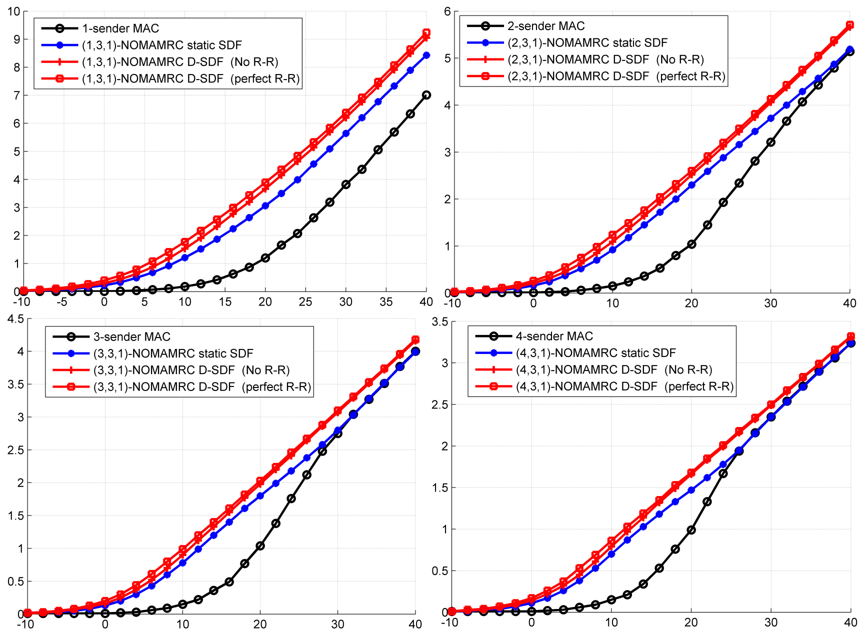


Figure 4.4: $R_\epsilon^{ind}(\boldsymbol{\gamma})$ of *waiting for all* D-SDF vs static SDF in $(M,3,1)$-MAMRC, where $M = 1,2,3,4$, $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma$.

set D-SDF achieve a better performance than *waiting for all* D-SDF. This is because at low SNR the delay of waiting for the correct decoding of all the sources prevents the D-SDF relays from cooperating;

- At high SNR, the *waiting for all* D-SDF and static SDF provide the same diversity order equal to $L+1$, but *waiting for all* D-SDF has a better coding gain, which could be further improved if $\gamma_{r,\tilde{r}} > 0$;

- At high SNR, both *waiting for at least one* D-SDF and *waiting for a least a specific subset* D-SDF start becoming suboptimal in terms of diversity order. To solve this problem, we conjecture that the relay should stay in the *waiting for all* mode at least $\Delta_{min} \leq \Delta$ channel uses before switching to other selection modes. Yet, this need to be investigated and confirmed in future work.

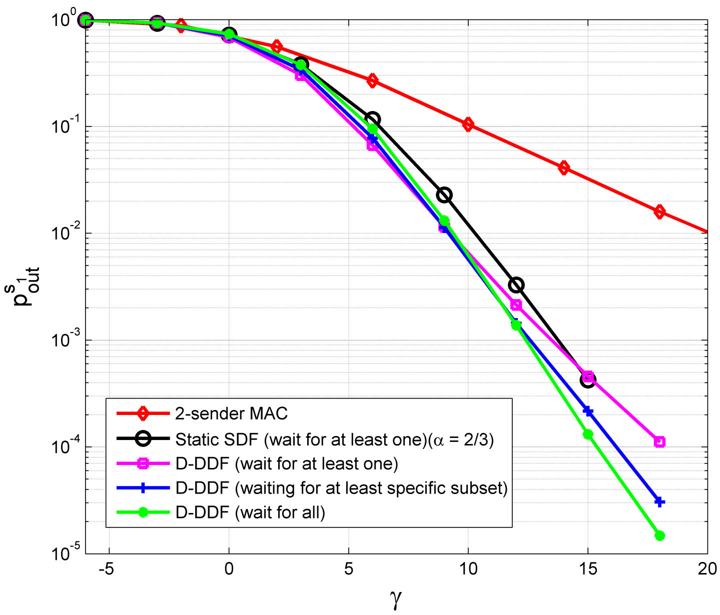


Figure 4.5: p^{ind}_{out} for an $(2,2,1)$-MAMRC, where $m_d = m_r = 1$, $\gamma_{r,d} = \gamma_{s,d} = \gamma$, $\gamma_{s,r} = 2\gamma$, and $R = 1(b.c.u)$.

4.4 Joint Network Channel Coding and Decoding

In this section, we explain our JNC distributed coding and decoding approach, detailing the structure of the encoders, when and how JNC decoding is performed.

4.4.1 Coding at the sources

The sources' packets (information bits and checks of CRC codes) are binary vectors $\mathbf{u}_s \in \mathbb{F}_2^K$ of length K. Each source basically employs a Bit Interleaved Coded Modulation (BICM) [127]. Binary vectors are first encoded with linear binary codes $C_s : \mathbb{F}_2^K \longrightarrow \mathbb{F}_2^{n_s}$ into binary codewords $\mathbf{c}_s \in \mathbb{F}_2^{n_s}$. For C_s we choose a puncturing systematic turbo codes of mother coding rate $w_s = K/n_s$. Each C_s consists of: (1) Two Recursive Systematic Convolutional (RSC) encoders with generator matrix $\mathbf{G}_s(D)$, concatenated in parallel using optimized semi-random interleaver π; (2) A puncturing process that determine the rate w_s of the turbo code and generates the turbo codeword $\mathbf{c}_s \in \mathbb{F}_2^{n_s}$. The latter is partitioned into T blocks using a partition function $p_s \in \mathbb{F}_2^{n_s}$. The block $\mathbf{c}_s^{(i)}$ sent in time slot $i \in \{1,\ldots,T\}$ is bit-interleaved into $\mathbf{b}_s^{(i)} \in \mathbb{F}_2^{n_{s,i}}$ using $\pi_{s,i}$. Finally, the interleaved binary sequence $\mathbf{b}_s^{(i)}$

is mapped by a memoryless modulator (i.e., $\phi_s : \mathbb{F}_2^{q_s} \to \mathcal{X}_s$) to the modulated sequence $\mathbf{x}_s^{(i)} \in \mathcal{X}_s^{N_i}$.

4.4.2 Decoding and re-encoding at the relays

At the end of each time slot $i \in \{1, \ldots, T-1\}$, the relays try to decode the sources' packets. The detailed description of the decoder is given in Sec. 4.4.3.

4.4.2.1 Linear network coding

When a relay r decides to transmit, it applies a local network coding function $F_r : \mathbb{F}_2^{K \times |\mathcal{S}_r|} \mapsto \mathbb{F}_2^K$ to generate its network coded packet. A convenient network coding function (see [97, Section III.C] for details) is

$$\mathbf{u}_r = \Pi_r^{-1} \left(\sum_{s \in \mathcal{S}_r} \oplus \mathbf{G}_{s,r} \Pi_{s,r}(\mathbf{u}_s) \right) \tag{4.19}$$

where $\sum \oplus$ represents the sum operator in $\mathbb{F}_2$, and $\mathbf{G}_{s,r}$ is a $K \times K$ binary matrix. In (4.19), $\Pi_{s,r}$ and Π_r (resp. Π_r^{-1}) denote independent pseudo-random interleavers (resp. deinterleavers) of length K. They are used to limit the detrimental effect of short cycles in the sum-product algorithm. Let $\tilde{\mathbf{u}}_{s,r} = \Pi_{s,r}(\mathbf{u}_s)$ and $\tilde{\mathbf{u}}_r = \Pi_r(\mathbf{u}_r)$ be the interleaved versions of the packets of the sources and the relay network coded packet, respectively. In Galois Field Network Coding (GFNC), the matrix $\mathbf{G}_{s,r}$ has the form $\mathbf{G}_{s,r} = \mathbf{I}_{K/q} \otimes \mathbf{G}(\alpha_{s,r})$ where $\mathbf{G}(\alpha_{s,r})$ is a $q \times q$ binary matrix which depends on the network coding coefficient $\alpha_{s,r} \in \mathbb{F}_{2^q}$. If the network coding coefficients are chosen from MDS codes, then GFNC achieves the full diversity order. BI-XOR based network coding, initially proposed in [114], is an alternative to GFNC. Each relay r generates its network coded packet using (4.19) by replacing $\mathbf{G}_{s,r}$ by $\mathbf{I}_K$. BI-XOR based network coding has the flexibility and simplicity of (linear) random network codes. Furthermore, BI-XOR based network coding can achieve a diversity order close to the full diversity order with high probability [114, Theorem 5]. The separate functions F_r, applied locally at each relay r, can be seen as components of a global function denoted by F_{nc}, referred to as global network coding function.

4.4.2.2 Channel coding

Binary network coded packets are encoded with linear binary codes $C_r : \mathbb{F}_2^K \mapsto \mathbb{F}_2^{n_r}$ into binary codewords $\mathbf{c}_r \in \mathbb{F}_2^{n_r}$. C_r consists of a rate-1/2 RSC encoder with generator matrix $\mathbf{G}_r(D)$. A partition function p_r determines the part $\mathbf{c}_r^{(i)}$ of $\mathbf{c}_r$ that will be sent in each time slot $i \in \{t_r, \ldots, T\}$. The resulting blocks $\mathbf{c}_r^{(i)}$ are then bit-interleaved into $\mathbf{b}_r^{(i)} \in \mathbb{F}_2^{n_{r,i}}$ using $\pi_{r,i}$. The interleaved binary codeword $\mathbf{b}_r^{(i)}$ is mapped by a memoryless modulator $\phi_r : \mathbb{F}_2^{q_r} \mapsto \mathcal{X}_r$ to the modulated sequence $\mathbf{x}_r^{(i)} \in \mathcal{X}_r^{N_i}$.

4.4.3 Joint network channel decoding at the destination

Both the relays and the destination use JNCD. Each relay activates its decoding process at the beginning of each time slot $i \in \{2, \ldots, T\}$, processing the received samples $[\mathbf{Y}_r^{(1)} \ldots \mathbf{Y}_r^{(i-1)}]$ and taking into consideration the cooperation modes of the $ind(r) - 1$ relays (we assume that the processing time of JNCD at the relays is fast and entails no delay). The destination starts decoding at the end of time slot T, processing the received samples $\mathbf{Y}_d$ (see (4.1)), and taking into consideration the cooperation modes (side information) of the L relays. Due to space limitation, we only describe the decoding at the destination. The maximum a posteriori decoding rule is expressed as

$$\hat{u}_{s,k} = \arg \max_{u_{s,k} \in \mathbb{F}_2} P(u_{s,k} | \mathbf{Y}_d, B)$$

where

$$\begin{aligned}
P(u_{s,k}|\mathbf{Y}_d, B) \propto & \sum_{\sim\{u_{s,k}\}} p(\underline{\mathbf{u}}_{\mathcal{T}_T}, \underline{\mathbf{c}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{c}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{b}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{b}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{x}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{x}}_{\mathcal{T}_T}^{(T)}, \mathbf{Y}_d) \\
& \times \Xi_B(\underline{\mathbf{u}}_{\mathcal{T}_T}, \underline{\mathbf{c}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{c}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{b}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{b}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{x}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{x}}_{\mathcal{T}_T}^{(T)}) \\
\overset{(a)}{=} & \sum_{\sim\{u_{s,k}\}} p(\mathbf{Y}_d^{(1)}, \ldots, \mathbf{Y}_d^{(T)} | \underline{\mathbf{x}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{x}}_{\mathcal{T}_T}^{(T)}) \\
& \times \Xi_B(\underline{\mathbf{u}}_{\mathcal{T}_T}, \underline{\mathbf{c}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{c}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{b}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{b}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{x}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{x}}_{\mathcal{T}_T}^{(T)}) \\
\overset{(b)}{=} & \sum_{\sim\{u_{s,k}\}} \left(\prod_{i=1}^{T} p(\mathbf{Y}_d^{(i)} | \underline{\mathbf{x}}_{\mathcal{T}_i}^{(i)}) \right) \\
& \times \Xi_B(\underline{\mathbf{u}}_{\mathcal{T}_T}, \underline{\mathbf{c}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{c}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{b}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{b}}_{\mathcal{T}_T}^{(T)}, \underline{\mathbf{x}}_{\mathcal{T}_1}^{(1)}, \ldots, \underline{\mathbf{x}}_{\mathcal{T}_T}^{(T)})
\end{aligned} \tag{4.20}$$

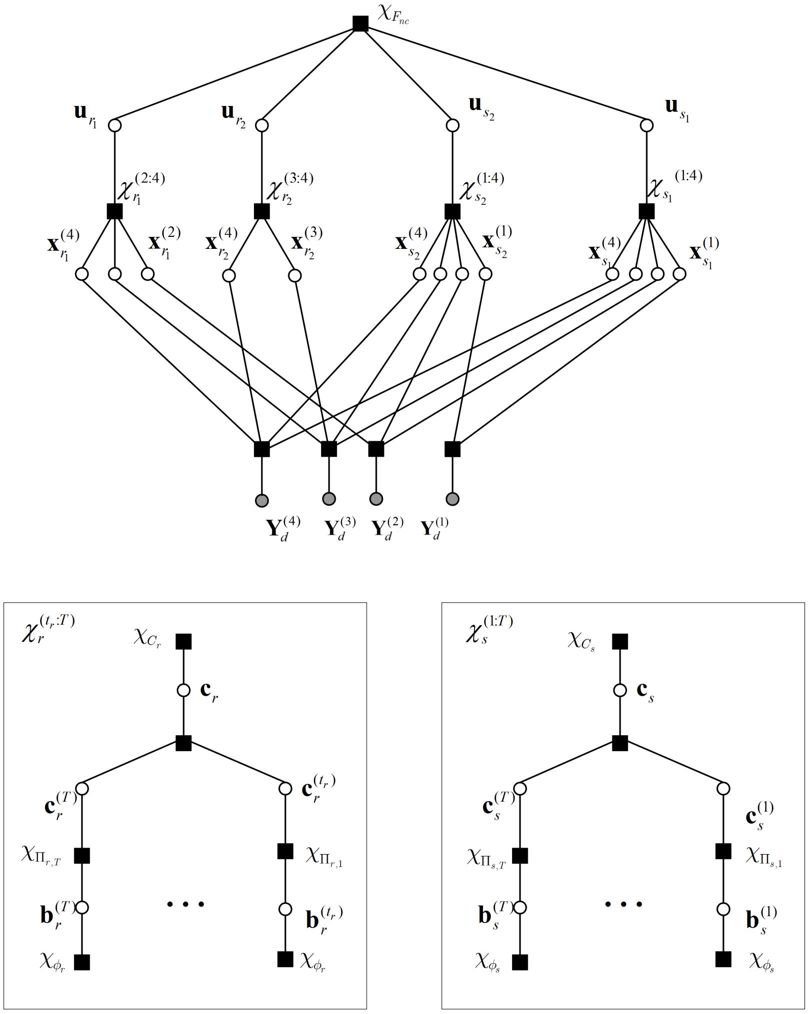


Figure 4.6: Factor graph representing the factorization in (4.20) and (4.22) for a $(2, 2, 1)$-NOMAMRC where $T = 4$, $t_{r_1} = 2$, and $t_{r_2} = 3$.

where Ξ_B is the characteristic function for B, the behavior of JNCC seen at the destination, that captures the relationship between the variables of the system. (a) follows from Bayes rule and the assumption that sources' packets have uniform priors. (b) follows from the system model assumptions detailed in 4.1. The characteristic function Ξ_B can be factorized as

$$\Xi_B(\underline{\mathbf{u}}_{\mathcal{T}_T}, \underline{\mathbf{c}}^{(1)}_{\mathcal{T}_1}, \ldots, \underline{\mathbf{c}}^{(T)}_{\mathcal{T}_T}, \underline{\mathbf{b}}^{(1)}_{\mathcal{T}_1}, \ldots, \underline{\mathbf{b}}^{(T)}_{\mathcal{T}_T}, \underline{\mathbf{x}}^{(1)}_{\mathcal{T}_1}, \ldots, \underline{\mathbf{x}}^{(T)}_{\mathcal{T}_T})$$

$$= \prod_{i=1}^{T} \left[\prod_{a \in \mathcal{T}_i} \left(\underbrace{\Xi_{\phi_a}(\mathbf{b}^{(i)}_a, \mathbf{x}^{(i)}_a)}_{=\mathbb{1}_{\{\mathbf{x}^{(i)}_a = \phi_a(\mathbf{b}^{(i)}_a)\}}} \underbrace{\Xi_{\pi_{a,i}}(\mathbf{c}^{(i)}_a, \mathbf{b}^{(i)}_a)}_{=\mathbb{1}_{\{\mathbf{b}^{(i)}_a = \pi_{a,i}(\mathbf{c}^{(i)}_a)\}}} \right) \right]$$

$$\times \prod_{s \in \mathcal{S}} \underbrace{\Xi_{p_s}(\mathbf{c}_s, \mathbf{c}^{(1)}_s, \ldots, \mathbf{c}^{(T)}_s)}_{=\mathbb{1}_{\{(\mathbf{c}^{(1)}_s, \ldots, \mathbf{c}^{(T)}_s) = p_s(\mathbf{c}_s)\}}} \prod_{r \in \mathcal{R}_T} \underbrace{\Xi_{p_r}(\mathbf{c}_r, \mathbf{c}^{(t_r)}_r, \ldots, \mathbf{c}^{(T)}_r)}_{=\mathbb{1}_{\{(\mathbf{c}^{(t_r)}_r, \ldots, \mathbf{c}^{(T)}_r) = p_r(\mathbf{c}_r)\}}}$$

$$\times \prod_{a \in \mathcal{T}_T} \underbrace{\Xi_{C_a}(\mathbf{u}_a, \mathbf{c}_a)}_{=\mathbb{1}_{\{\mathbf{c}_a = C_a(\mathbf{u}_a)\}}} \Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{T}_T}) \tag{4.21}$$

where $\Xi_{F_{nc}}$, Ξ_{C_a}, Ξ_{p_a}, $\Xi_{\pi_{a,i}}$, and Ξ_{ϕ_a} represent the characteristic functions of the behavioral modeling of the global network encoder, the channel encoder, the partition function, the channel interleavers, and the modulator of any node $a \in \{s, r\}$. The characteristic function $\Xi_{F_{nc}}$ can be factorized as

$$\Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}_T}) = \prod_{r \in \mathcal{R}_T} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r), \tag{4.22}$$

where $\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r)$ represents the characteristic function of the behavioral modeling of the local network coding function given by (4.19). Finally, the characteristic function Ξ_{F_r} can be factorized as

$$\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r) = \prod_{s \in \mathcal{S}_r} \underbrace{\Xi_{\Pi_{s,r}}(\mathbf{u}_s, \tilde{\mathbf{u}}_{s,r})}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_{s,r} = \Pi_{s,r}(\mathbf{u}_s)\}}} \underbrace{\Xi_{\Pi_r}(\mathbf{u}_r, \tilde{\mathbf{u}}_r)}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_r = \Pi_r(\mathbf{u}_r)\}}} \times \underbrace{\Xi_{\tilde{F}_r}(\tilde{\underline{\mathbf{u}}}_{\mathcal{S}_r}, \tilde{\mathbf{u}}_r)}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_r = \sum_{s \in \mathcal{S}_r} \oplus \mathbf{G}_{s,r} \tilde{\mathbf{u}}_{s,r}\}}} \tag{4.23}$$

Fig. 4.6 depicts the factor graph representing the above factorization a $(2,2,1)$-MAMRC where $T = 4$, $t_{r_1} = 2$, and $t_{r_2} = 3$. Fig. 4.7 shows the factor graph representing the factorization (4.22) and (4.23) for a $(2,2,1)$-MAMRC. A brute-force approach to evaluate the marginal a posteriori probability $P(u_{s,k}|\mathbf{Y}_d, B)$ in (4.20) is of course intractable. We thus resort to the sum-product algorithm [115]. Since the overall factor graph has cycles, the algorithm is applied *iteratively*. As well known, the message-passing schedule may impact the convergence. Once it is specified, messages along the edges connecting the different nodes of the factor graph circulate until convergence (to approximate marginals). In our case, each iteration comprises the following steps:

1. Detection and demapping: Compute the messages $\mu_{\Xi_{\phi_a} \to b^{(i)}_{a,k}}(b^{(i)}_{a,k})$, where $a \in \{s, r\}$, $i = 1, \ldots, T$, and $k = 1, \ldots, n_{a,i}$, using the channel observations $\mathbf{Y}^{(i)}_d$ and the messages of the variable nodes $\mu_{b^{(i)}_{a',k'} \to \Xi_{\phi_{a'}}}(b^{(i)}_{a',k'})$, $(a', k') \neq (a, k)$, and route them, through the interleaver connections and the partition functions, to the variables nodes $c^{(i)}_{a,j}$, where $j = 1, \ldots, n_a$.

2. Channel decoding of the sources: Compute the messages $\mu_{\Xi_{C_s} \to c^{(i)}_{s,k}}(c^{(i)}_{s,k})$ and $\mu_{\Xi_{C_s} \to u_{s,\ell}}(u_{s,\ell})$, where $\ell = 1, \ldots, K$.

3. Network decoding: Compute the messages $\mu_{\Xi_{F_{nc}} \to u_{r,\ell}}(u_{r,\ell})$.

4. Channel decoding of the relays: Compute the messages $\mu_{\Xi_{C_r} \to c^{(i)}_{r,k}}(c^{(i)}_{r,k})$, where $i = t_r, \ldots, T$, and $\mu_{\Xi_{C_r} \to u_{r,\ell}}(u_{r,\ell})$.

5. Network decoding: Compute the messages $\mu_{\Xi_{F_{nc}} \to u_{s,\ell}}(u_{s,\ell})$.

6. Based on the product of all incoming messages at the nodes $u_{a,\ell}$, where $a \in \{s, r\}$, and $\ell = 1, \ldots, K$, hard decisions are made to obtain the estimates $\hat{\mathbf{u}}_a$. Then, CRC checks are performed to extract the correctly decoded frames. Finally, separate network decoding is performed on the correctly decoded messages. For GFNC, decoding can be performed using Gauss-Jordan elimination. For BI-XOR-based or XOR-based network coding, if a network coded packet of a relay r and $|\mathcal{S}_r| - 1$ packets of the sources in $\mathcal{S}_r$ are correctly decoded then all the packets of $\mathcal{S}_r$ are correctly decoded.

7. If the M source packets are correctly decoded or if the maximum number of iterations is reached, the iterative process stops. Else another iteration is performed.

4.4.4 Network decoding

In this section, we detail the messages generated by the network decoder. The messages generated from $\Xi_{F_{nc}}$ to $u_{r,\ell}$, where $r \in \mathcal{R}_T$, and $\ell = 1, \dots, K$ are given by

$$\begin{aligned} \mu_{\Xi_{F_{nc}} \to u_{r,\ell}}(u_{r,\ell}) = & \sum_{\sim\{u_{r,\ell}\}} \Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}_T}) \\ \prod_{(r',\ell') \neq (r,\ell)} & \mu_{u_{r',\ell'} \to \Xi_{F_{nc}}}(u_{r',\ell'}) \prod_{s \in \mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'} \to \Xi_{F_{nc}}}(u_{s,\ell'}). \end{aligned} \tag{4.24}$$

We further simplify (4.24) by only considering the check equation provided by r (separately from the other relays). In this case, the messages generated from $\Xi_{F_{nc}}$ to $u_{r,\ell}$, are given by

$$\begin{aligned} \mu_{\Xi_{F_{nc}} \to u_{r,\ell}}(u_{r,\ell}) \approx \mu_{\Xi_{F_r} \to u_{r,\ell}}(u_{r,\ell}) = & \sum_{\sim\{u_{r,\ell}\}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r) \\ \prod_{\ell' \neq \ell} \mu_{u_{r,\ell'} \to \Xi_{F_r}}(u_{r,\ell'}) & \prod_{s \in \mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'} \to \Xi_{F_r}}(u_{s,\ell'}). \end{aligned} \tag{4.25}$$

Similarly, the messages generated from the check nodes Ξ_{F_r}, to $u_{s,\ell}$, where $s \in \mathcal{S}_r$, and $\ell = 1, \dots, K$, are given by

$$\begin{aligned} \mu_{\Xi_{F_r} \to u_{s,\ell}}(u_{s,\ell}) = & \sum_{\sim\{u_{s,\ell}\}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}_r}, \mathbf{u}_r) \\ & \prod_{\ell'=1}^{K} \mu_{u_{r,\ell'} \to \Xi_{F_r}}(u_{r,\ell'}) \prod_{(s',\ell') \neq (s,\ell)} \mu_{u_{s',\ell'} \to \Xi_{F_r}}(u_{s',\ell'}). \end{aligned} \tag{4.26}$$

4.4.5 Practical schemes evaluation

There are, of course, an infinity of MAMRC configurations (SNR distribution, transmission rate, number of sources, number of relays, etc.). By carefully selecting a few configurations as examples, our intention is to better understand how D-SDF performs. The quality of service is measured in terms of individual/common Block Error Rate (BLER). We choose $T = 7$. Each source packet has a length of $K = 480$ information bits. The sources employ

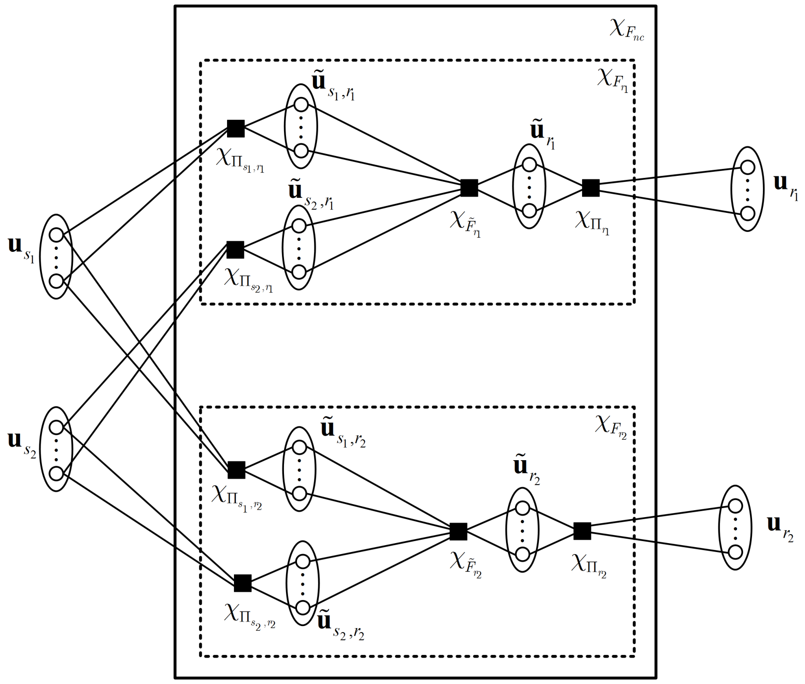


Figure 4.7: Factor graph representing the factorization (4.22) and (4.23) for a $(2, 2, 1)$-NOMAMRC when $\mathcal{R}_T = \mathcal{R}$, and $\mathcal{S}_r = \mathcal{S}, \forall r \in \mathcal{R}$.

identical punctured turbo codes of coding rate $w_s = 1/3$ (mother codes) made of two 4-state rate-1/2 RSC encoders with generator matrix $\mathbf{G}_s(D) = \left[1 \; \frac{1+D^2}{1+D+D^2}\right]$. We design the partition function p_s such that $n_{s,1} = K$ (only the systematic bits are transmitted in the first block) and $n_{s,i} = K(1/\omega_s - 1)/(T-1) = K/3$, $\forall i \in \{2, \dots, T\}$, where the parity bits are selected in a round robin fashion one bit from each RSC encoder output. The relays employ 8-state rate-1/2 RSC encoders with generator matrix $\mathbf{G}_r(D) = \left[1 \; \frac{1+D^3}{1+D+D^2+D^3}\right]$. The proposed constituent encoders are given only as examples to illustrate the benefits of our approach. We design the selecting vector p_r such that $n_{r,i} = n_{s,i}$, $\forall i \in \{2, \dots, T\}$, where each block contains either parity bits or systematic bits. 4-QAM with Gray labeling is assumed at the sources and relays ($q_s = q_r = 2$). Six iterations of the SPA are performed at the relays (imperfect S-R and Relay-to-Relay (R-R) links), and ten at the destination. Static SDF is used as a benchmark [94, 103].

In the first set of simulations, we look at a $(2, 2, 1)$-NOMAMRC where each relay applies BI-XOR based network coding. First, we aim at evaluating the advantage of D-SDF over static SDF regardless of the selection strategy. Hence, we consider perfect S-R links (there is no R-R links since all the relays will be activated at the same time). Fig. 4.8 shows the results. We observe that the proposed coding schemes for D-SDF achieve the same diversity

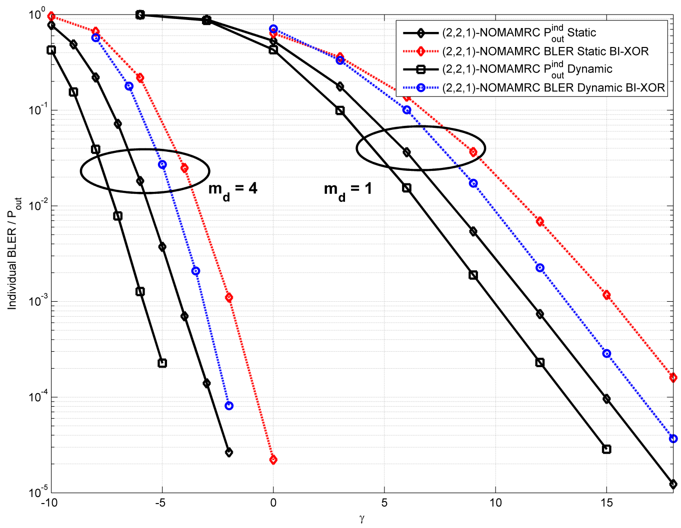


Figure 4.8: IBLER and p_{out}^{ind} of $(2,2,1)$-NOMAMRC, where $R = 2/3$ (b./c.u), $\gamma_{s,d} = \gamma_{r,d} = \gamma$, $\gamma_{s,r} = \infty$. For static SDF, we chose $\alpha = 2/3$.

order as the one proposed for static SDF and provide additional coding gain, up to 2.5 dB when $m_d = 1$ in this scenario. Secondly, we consider non-perfect S-R links and that the relays are working in *Waiting for all* mode. Fig. 4.9 shows the results. The previous observations are reconfirmed but the coding gain is about 1.5 dB in this scenario.

In the second set of simulations, we consider a $(2,1,1)$-NOMAMRC with asymmetric imperfect S-R links. We choose $\gamma_{s_1,r} = \gamma + 3\text{dB}$, $\gamma_{s_2,r} = \gamma - 3\text{dB}$, and $\gamma_{r,d} = \gamma_{s,d}$. We consider *threshold selection* D-SDF and we test different values of $T_{th,r}$. Fig. 4.10 shows the results. We observe that small values of $T_{th,r}$ are a good choice for s_1, but a bad choice for s_2, and the opposite is true for high values of $T_{th,r}$. The choice $T_{th,r} = 4$ leads to the best common BLER among the investigated thresholds.

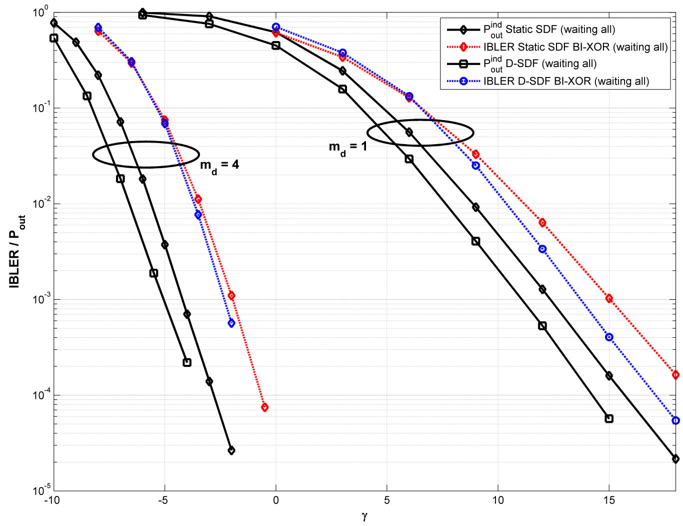


Figure 4.9: IBLER and p_{out}^{ind} of $(2, 2, 1)$-NOMAMRC, where $R = 2/3$ (b./c.u), $\gamma_{s,d} = \gamma_{r,d} = \gamma_{s,r} = \gamma_{r,\tilde{r}} = \gamma$. For static SDF, we chose $\alpha = 2/3$.

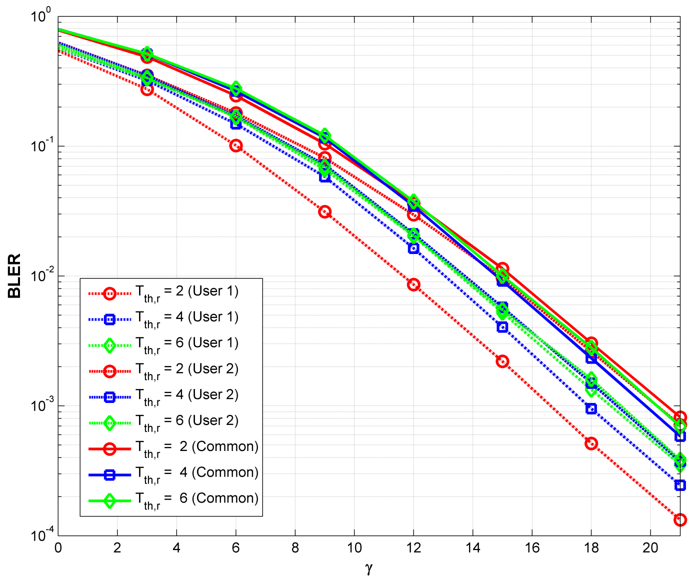


Figure 4.10: Individual and common BLER of $(2, 1, 1)$-NOMAMRC, where $\gamma_{s,d} = \gamma_{r,d} = \gamma_{s,r} = \gamma$, $T_{th,r} = 2, 4, 6$.

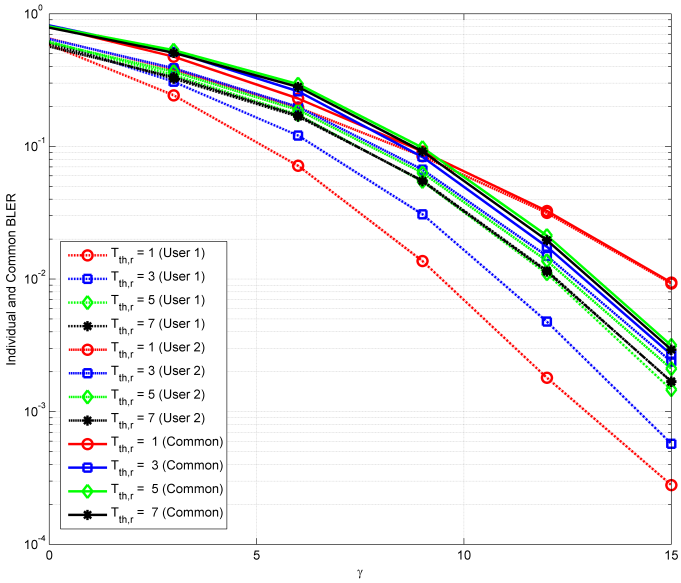


Figure 4.11: Individual and common BLER of $(2, 2, 1)$-NOMAMRC, where $\gamma_{s,d} = \gamma_{r,d} = \gamma_{s,r} = \gamma$, $T_{th,r} = 1, 3, 5, 7$.

CHAPTER 5

Static SDF for MAMRC with Limited Feedback

In this Chapter, we investigate cooperative Incremental Redundancy Hybrid-ARQ (IR-HARQ) strategies based on Selective Decode and Forward (SDF) relaying for the slow fading Orthogonal Multiple Access Multiple Relay Channel (OMAMRC), defined in sec. 3.1.1. Limited feedback from the destination to the relays and the sources is allowed. The destination uses feedback messages to control the (re)transmission of the different nodes (relays and/or sources) with the aim of improving both the spectral efficiency and the reliability (increasing the possibility of decoding all the packets of the sources).

5.1 System model

We consider a time-slotted OMAMRC where a set of statistically independent sources $\mathcal{S} = \{s_1, \ldots, s_M\}$, want to communicate their packets (messages), i.e., $\mathbf{u}_s \in \mathbb{F}_2^K$ of K information bits, to a common destination d with the help of a set of relays $\mathcal{R} = \{r_1, \ldots, r_L\}$. A frame is made of the time slots that are used for the transmission of the M sources' packets. The maximum frame duration is $M + T_{max}$ time slots where $T_{max} \geq L$ is a system design parameter. Transmissions within a frame are divided into two phases. The first phase consists of M time slots, in which each source $s \in \mathcal{S}$ takes turn to transmit its packet. Each time slot has a duration of N_1 channel uses. The second phase consists of $t \in \{1, \ldots, T_{max}\}$ time slots (rounds). Each round has a duration of N_2 channel uses. The destination decides the number of rounds that will be used in the second phase and the node that should transmit in each round. Hence, limited feedback messages are used and assumed perfect. Each node is equipped with one antenna. All sources are symmetric, i.e., have the same transmission rate, power, and priority. Channel State Information (CSI) is available only at the receiver of each direct link and is assumed perfect. The set of correctly decoded packets

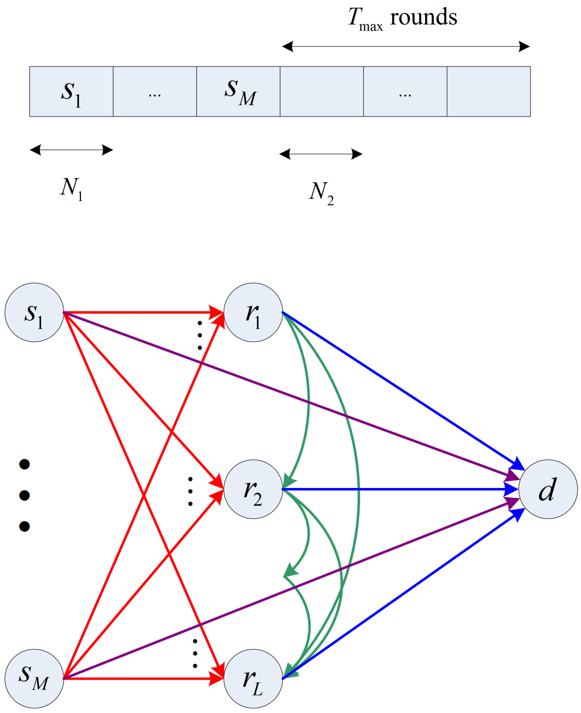


Figure 5.1: $(M, L, 1)$-OMAMRC model with time-division based orthogonal multiple access

at node $b \in \mathcal{R} \cup \{d\}$ at the end of round $t \in \{0, \ldots, T_{max}\}$ is denoted by $\mathcal{S}_{b,t} \subseteq \mathcal{S}$ and referred hereafter as the decoding set of node b at round t. Note that the end of the first transmission phase is considered as the end of round zero. For the sake of notation simplicity, we use the convention that $\mathcal{S}_{s,t} = \{s\}$, $\forall l \in \{0, \ldots, T_{max}\}$. The radio-links between the different nodes are assumed fixed within a frame transmission but changes independently from frame to frame (slow fading/quasi-static assumption). In the following, $y_{a,b,k}$ denotes the base-band signal transmitted from node $a \in \mathcal{S} \cup \mathcal{R}$ and received at node $b \in \mathcal{R} \cup \{d\}$ for channel use k, $h_{a,b}$ the static channel between a and b, $n_{a,b,k}$ the AWGN noise sample, and $x_{a,k} \in \mathbb{C}$ the associated carried modulated symbol. In the first phase, during the time slot assigned to the source node s, the received signal at node $b \in \mathcal{R} \cup \{d\}$ can be written as

$$y_{s,b,k} = h_{s,b} x_{s,k} + n_{s,b,k}, \tag{5.1}$$

where $k = 1, \ldots, N_1$. In the second phase, during round $t = 1, \ldots, T_{max}$, the received signal at node $b \in \mathcal{R} \cup \{d\} \backslash \{\hat{a}_t\}$, where $\hat{a}_t \in \mathcal{S} \cup \mathcal{R}$ is the selected node to transmit during round t, can be written as

$$y_{\hat{a}_t,b,k} = h_{\hat{a}_t,b} x_{\hat{a}_t,k} + n_{\hat{a}_t,b,k}, \tag{5.2}$$

where $k = 1, \ldots, N_2$. Thanks to the orthogonal channel access in time of the relays, the relays help each other, i.e., each relay when not transmitting listens to the active relay transmission in order to increase its decoding set cardinality. Thus, each non-active relay behaves as a destination node in its decoding process. The channel fading coefficients $h_{a,b}$ are assumed independent and follow a circularly complex Gaussian probability distribution function of mean 0 and variance $\gamma_{a,b}$ denoted $\mathcal{CN}(0, \gamma_{a,b})$ while the additive AWGN noise samples $n_{a,b,k}$ are i.i.d and follow the pdf $\mathcal{CN}(0, 1)$. We further assume that the transmitted symbols' power (per complex dimension) from the sources and the relays are normalized to unity. $\gamma_{a,b}$ is the average received power at the receiver b from transmitter a. Shadowing and the path-loss can be included in $\gamma_{a,b}$.

5.2 Problem formulation

The destination knows perfectly $\mathbf{h} = [h_{s_1,d}, \ldots, h_{s_M,d}, h_{r_1,d}, \ldots, h_{r_L,d}]$, the CSI of sources-to-destination (S-D) and relays-to-destination (R-D) links. However, the destination lacks the knowledge of the CSI of sources-to-relays (S-R) and relays-to-relays (R-R) links which prevents any global brute force optimization that can determine the best sequence of nodes to activate in consecutive rounds in order to maximize the throughput, i.e., minimize the average transmission frame duration and maximize rate of correctly decoded messages. Yet, each relay r at the beginning of round t sends its decoding set $\mathcal{S}_{r,t-1}$ to the destination which can be viewed as a partial CSI on the S-R and R-R links. The destination selects (by broadcasting a control messages) the node $\hat{a}_t$ that transmits during round t together with the subset $\hat{\mathcal{S}}_t$ of sources (belonging to the selected node decoding set) it has to collaborate with. This selection depends on the destination's knowledge of $\mathbf{h}$ and the set $\mathcal{P}_{t-1}$ that gathers $\{(\hat{a}_1, \hat{\mathcal{S}}_1), \cdots, (\hat{a}_{t-1}, \hat{\mathcal{S}}_{t-1})\}$, $\{\mathcal{S}_{r,t-1}, \forall r \in \mathcal{R}\}$ and $\mathcal{S}_{d,t-1}$. It comes naturally that $\mathcal{P}_0$ gathers only the subsets $\{\mathcal{S}_{b,0}, \forall b \in \mathcal{R} \cup \{d\}\}$. Let $\mathtt{E}_t(a_t, \mathcal{S}_t | \mathbf{h}, \mathcal{P}_{t-1})$ be the destination event that at least one source is not decoded correctly at the end of round t with respect to $a_t \in \mathcal{R} \cup \mathcal{S}$ being the active node, $\mathcal{S}_t \subseteq \mathcal{S}_{a_t,t-1}$ the subset of sources that the active node a_t will help, and conditional on the knowledge of $\mathbf{h}$ and $\mathcal{P}_{t-1}$. We define similarly $\mathtt{O}_{s,t}(a_t, \mathcal{S}_t | \mathbf{h}, \mathcal{P}_{t-1})$ the destination event that source s is not decoded correctly. For any event $\mathtt{A}_t(\hat{a}_t, \hat{\mathcal{S}}_t | \mathbf{h}, \mathcal{P}_{t-1})$, where $\mathtt{A}_t \in \{\mathtt{E}_t, \mathtt{O}_{s,t}\}$, the probability $\Pr\{\mathtt{A}_t\}$ is associated which can be formally defined as $\mathbb{E}(\mathbb{1}_{\{\mathtt{A}_t(\hat{a}_t, \hat{\mathcal{S}}_t | \mathbf{h}, \mathcal{P}_{t-1})\}})$ where $\mathbb{E}(.)$ is the expectation operator, and $\mathbb{1}_{\{\mathtt{A}_t(\hat{a}_t, \hat{\mathcal{S}}_t | \mathbf{h}, \mathcal{P}_{t-1})\}}$ takes the value 1 if $\mathtt{A}_t(\hat{a}_t, \hat{\mathcal{S}}_t | \mathbf{h}, \mathcal{P}_{t-1})$ is true, 0 otherwise.

The average number of retransmissions rounds, represented by the random variable T, can be expressed as

$$
\begin{aligned}
\mathbb{E}(T) &= \sum_{t=1}^{T_{max}} t \Pr\{T = t\} \\
&= \sum_{t=1}^{T_{max}} t \Pr\{\mathtt{E}_{t-1} \cap \bar{\mathtt{E}}_t\} + T_{max} \Pr\{\mathtt{E}_{T_{max}}\} \\
&= \sum_{t=1}^{T_{max}-1} \Pr\{\mathtt{E}_t\}.
\end{aligned}
\tag{5.3}
$$

The maximum and the minimum transmission rate (in bits per channel use (b.c.u)) are defined as follow $R_{max} = K/N_1$, $R_{min} = MR_{max}/(M + T_{max}\alpha)$, where $\alpha = N_2/N_1$. The average transmission rate $\bar{R}$ can be expressed as

$$
\bar{R} = \frac{MR_{max}}{M + \alpha\mathbb{E}(T)}. \tag{5.4}
$$

The expected number of received information bits during each frame is given by $\sum_{s\in\mathcal{S}} K(1-\Pr\{\mathtt{O}_{s,T_{max}}\})$, Hence, the throughput η can be defined as (see [107])

$$
\eta = \frac{1}{M}\bar{R}\sum_{s\in\mathcal{S}}(1 - \Pr\{\mathtt{O}_{s,T_{max}}\}). \tag{5.5}
$$

We distinguish, hereafter, two distributed encoding and decoding frameworks. In the following we will omit the dependency on $\mathbf{h}$, and $\mathcal{P}_{t-1}$ for the sake of conciseness. $I_{a,b}$ denotes the instantaneous mutual information between the transmitting node $a \in \mathcal{S} \cup \mathcal{R}$, and the receiving node $b \in \mathcal{R} \cup \{d\}$.

5.2.1 Distributed Channel Coding/Joint Distributed Channel Decoding (DCC/JDCC) framework

In DCC/JDCD, no network coding is performed at the relays, the transmitted sequence of an activated relay is a concatenation of separate sequences, each one corresponds to a correctly decoded source and adds extra redundancy to its packet. In this case, we can write the common outage event as

$$\mathtt{E}_t(a_t, \mathcal{S}_t) = \Big\{ R_{max} > I_t^c(a_t, \mathcal{S}_t) \Big\}, \tag{5.6}$$

where

$$I_t^c(a_t, \mathcal{S}_t) = \min_{s \in \bar{\mathcal{S}}_{d,t-1}} \Big(I_{s,d} + \sum_{l=1}^{t-1} \frac{\alpha}{|\hat{\mathcal{S}}_l|} I_{\hat{a}_l,d} \mathbb{1}_{\{s \in \hat{\mathcal{S}}_l\}} + \frac{\alpha}{|\mathcal{S}_t|} I_{a_t,d} \mathbb{1}_{\{s \in \mathcal{S}_t\}} \Big), \tag{5.7}$$

where $\bar{\mathcal{S}}_{d,t-1} = \mathcal{S} \backslash \mathcal{S}_{d,t-1}$. The individual outage event can be expressed similarly as

$$\mathtt{O}_{s,t}(a_t, \mathcal{S}_t) = \Big\{ R_{max} > I_t^s(a_t, \mathcal{S}_t) \Big\}, \tag{5.8}$$

where

$$I_t^s(a_t, \mathcal{S}_t) = I_{s,d} + \sum_{l=1}^{t-1} \frac{\alpha}{|\hat{\mathcal{S}}_l|} I_{\hat{a}_l,d} \mathbb{1}_{\{s \in \hat{\mathcal{S}}_l\}} + \frac{\alpha}{|\mathcal{S}_t|} I_{a_t,d} \mathbb{1}_{\{s \in \mathcal{S}_t\}}. \tag{5.9}$$

5.2.2 Joint Network Channel Coding/Decoding (JNCC/JNCD) framework

In JNCC/JNCD, the transmitted sequence of an activated node $\hat{a}_\ell$, where $\ell \in \{1, \ldots, t-1\}$ and the transmitted sequences of the sources in $\hat{\mathcal{S}}_\ell$ form a joint codeword on the messages of the sources $\hat{\mathcal{S}}_\ell$. It is also the assumption for the transmitted sequence of the candidate node a_t with respect to the helped source subset S_t. In this case, $I_t^c(a_t, \mathcal{S}_t)$ and $I_t^s(a_t, \mathcal{S}_t)$ can be written as (see [93]),

$$I_t^c(a_t, \mathcal{S}_t) = \min_{\mathcal{U} \subseteq \bar{\mathcal{S}}_{d,t-1}} \frac{1}{|\mathcal{U}|} \Big(\sum_{s \in \mathcal{U}} I_{s,d} + \sum_{l=1}^{t-1} \alpha I_{\hat{a}_l,d} \mathbb{1}_{\{\mathcal{U} \cap \hat{\mathcal{S}}_l \neq \emptyset\}} + \alpha I_{a_t,d} \mathbb{1}_{\{\mathcal{U} \cap \mathcal{S}_t \neq \emptyset\}} \Big), \tag{5.10}$$

and

$$I_t^s(a_t, \mathcal{S}_t) = \max_{\mathcal{I} \subseteq \bar{\mathcal{S}}_{d,t-1}} \min_{\mathcal{U} \subseteq \bar{\mathcal{I}}: s \in \mathcal{U}} \frac{1}{|\mathcal{U}|} \Big(\sum_{s \in \mathcal{U}} I_{s,d} + \sum_{l=1}^{t-1} \alpha I_{\hat{a}_l,d} \mathbb{1}_{\{\hat{C}_{l,s}\}} + \alpha I_{a_t,d} \mathbb{1}_{\{C_{t,s}\}} \Big), \tag{5.11}$$

where $\bar{\mathcal{I}} = \bar{\mathcal{S}}_{d,t-1} \setminus \mathcal{I}$ and

$$\begin{aligned} \hat{C}_{l,s} &= \Big\{ \{s \in \hat{\mathcal{S}}_l \cap \mathcal{U}\} \text{and} \{\hat{\mathcal{S}}_l \cap \mathcal{I} = \emptyset\} \Big\}, \\ C_{t,s} &= \Big\{ \{s \in S_t \cap \mathcal{U}\} \text{and} \{S_t \cap \mathcal{I} = \emptyset\} \Big\}. \end{aligned}$$

5.3 Feedback strategy overview

In the first phase (M time-slots), the sources broadcast their messages to the destination and the L relays. In OMAMRC without feedback, the second phase is dedicated only to the transmission of the relays. Each SDF relay is assigned once and for all a predefined exclusive set of rounds to cooperate with its decoding set (see [93, 97]). The simplest strategy with limited feedback is referred to as OMAMRC with ACK/NACK. The only difference with respect to OMAMRC without feedback is that the destination tries to decode at the end of the first phase and at the end of each round in the second phase. Based on CRC checks, the destination either broadcasts a common ACK message to the sources and the relays if all the messages are decoded correctly or a common NACK. The broadcasting of the common ACK message indicates the end of the current frame. In the following strategies we assume that each relay use in-band signaling channels to forwards to the destination its decoding set (or updates of the decoding set if changed) prior to any transmission round such that the destination can select the next node to transmit following a certain criterion, e.g., minimization of the common outage probability. When the destination has successfully decoded all the sources messages at a given round, the relays and sources start a new transmission frame as in the case of OMAMRC with ACK/NACK. It is clear from Section 5.2 that maximizing $I_t^c(a_t, \mathcal{S}_t)$ for a certain channel outcome $\mathbf{h}$, and conditional on $\mathcal{P}_{t-1}$ minimizes the common outage probability $\Pr\{\mathtt{E}_t\}$ which is our practical approach to maximize the transmission rate defined in (5.4). Since $\Pr\{\mathtt{O}_{s,T_{max}}\} \leq \Pr\{\mathtt{E}_{T_{max}}\}$ for all $s \in \mathcal{S}$, $\Pr\{\mathtt{E}_t\} \leq \Pr\{\mathtt{E}_{t-1}\}$, and the sources are symmetric, we can expect that this approach can improve the throughput given by (5.5) as well.

5.3.1 Strategy 1: OMAMRC with common ACK/NACK and node selection

At the beginning of each round $t = 1, \ldots, T_{max}$, if the destination does not correctly decode all the packets, it broadcasts a common NACK message. Upon the reception of the NACK, the relays forward to the destination an update about their decoding sets. Each node will cooperate with its decoding set. At the beginning of round t, the destination chooses the node $a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}$ that maximizes $I_t^c(a_t, \mathcal{S}_{a_t,t-1})$. In this case, the node selection process can be written as

$$\hat{a}_t = \arg \max_{a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}} I_t^c(a_t, \mathcal{S}_{a_t,t-1}), \tag{5.12}$$

where $\hat{\mathcal{S}}_t = \mathcal{S}_{\hat{a}_t,t-1}$. Upon the reception of a common ACK, a new frame transmission starts.

Remark: The number of feedback bits per round needed for this strategy is at most $c_1 = \lfloor \log_2(M+L) \rfloor + 1$ (bits).

5.3.2 Strategy 2: OMAMRC with common ACK/NACK, node and message subset selection

This strategy builds on Strategy 1. When the destination selects a relay to be active, it provides also a subset of the sources that a chosen relay has to help. In this case, the selection process can be written as

$$(\hat{a}_t, \hat{\mathcal{S}}_t) = \arg \max_{a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}, \mathcal{S}_t \subseteq \mathcal{S}_{a_t,t-1}} I_t^c(a_t, S_t). \tag{5.13}$$

Remark: The number of feedback bits per round needed for this strategy is at most $c_2 = \lfloor \log_2(M+L) \rfloor + 1 + M$ (bits).

5.3.3 Strategy 3: OMAMRC with individual ACK/NACK, and node selection

This strategy tries to reduce the needed feedback signaling for strategy 2. At the beginning of each round t, the destination sends an individual ACK (resp. NACK) to the sources that have been decoded correctly (resp. incorrectly). The relays and the sources by listening to these control channels can deduce the decoding set of the destination $\mathcal{S}_{d,t-1}$. If $\mathcal{S}_{d,t-1} = \mathcal{S}$ (all the messages have been decoded correctly at the destination) then a new frame transmission begins. Otherwise each relay forwards to the destination an update about their decoding sets. The selection process selects the node $a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}$ knowing that this selected node will cooperate with the sources in $\bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{a_t,t-1}$. In this case, the selection process can be written as

$$\hat{a}_t = \arg \max_{a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}} I_t^c(a_t, \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{a_t,t-1}). \tag{5.14}$$

where $\hat{\mathcal{S}}_t = \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{\hat{a}_t,t-1}$.

Remark: The number of feedback bits per round needed for this strategy is at most $c_3 = \lfloor \log_2(M+L) \rfloor + M$ (bits).

Remark: The in-band signaling required for strategies 1,2 and 3 is at most $M * L * T_{max}$ (bits). Contrary to OMAMRC without feedback and OMAMRC with ACK/NACK only which require $M * L$ (bits).

The strategy pseudo code is given in Algorithm 1.

Algorithm 1 : Selection process of strategy 3 at the beginning of round t

(Initialization)
$I_{max} = 0$
(Loop)
For For each node $a_t \in \mathcal{R} \cup \bar{\mathcal{S}}_{d,t-1}$ **Do**

- Calculate $I_t^c(a_t, \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{a_t,t-1})$ using (5.7) or (5.10).
- **If** $I_t^c(a_t, \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{a_t,t-1}) > I_{max}$ **Then**
  - $I_{max} = I_t^c(a_t, \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{a_t,t-1})$
  - $\hat{a}_t = a_t$

End For

5.3.4 Practical compensation

A practical analysis tool for the block fading environment is the outage probability [137], which for large block lengths, serves as a lower bound to the frame error rate of a practical coded transmission protocol. Hence, in (5.4) it is assumed that K, N_1, N_2 are big enough such that the effect of feedback, and in-band signaling, on the transmission rate is negligible. However, to be more realistic we assume that transmitting same amount of data in practice requires more channel uses than the optimal case (perfect and free feedback, and in-band signaling), hence we define the effective transmission rate $\bar{R}_e$ per source as

$$\bar{R}_e = \frac{R}{M + \alpha\beta\mathbb{E}(T)} = \bar{R}\frac{M + \alpha\mathbb{E}(T)}{M + \alpha\beta\mathbb{E}(T)}, \tag{5.15}$$

where $\beta \geq 1$ is the load effect compensating factor (load factor) and its value depends on both the feedback, and in-band signaling of the strategy. For example for strategy 3, we can write $\beta = 1 + \frac{c_3 + M*L}{C*N_2}$, where C is the capacity feedback channel and the in-band channel. A numerical result could clarify the idea so lets consider $C = 0.1$ bits/channel use (b/c.u) , $M = 3, L = 3, T_{max} = 3$, and $N_2 = 256$ channel uses, in this case, $\beta = 1.62$, which means that the feedback and the in-band signaling will cost 62% more resources than the optimal transmission. Finally, the effective throughput can be written as

$$\eta_e = \bar{R}_e \sum_{s \in \mathcal{S}} (1 - \Pr\{\mathtt{O}_{s,T_{max}}\}). \tag{5.16}$$

5.4 Numerical Results

In this section, we assume Gaussian i.i.d input distribution and $I_{a,b} = \log(1 + |h_{a,b}|^2)$. Note that different formula could be used for calculating $I_{a,b}$, without affecting the basic concept of this work. For example, see [138,139]. By arbitrarily choosing few configurations as examples, we want to evaluate the effectiveness and the relative performance among the different proposed $(M, L, 1)$-OMAMRC feedback cooperative strategies. We chose a $(3, 3, 1)$-OMAMRC, $T_{max} = 3$, $R = 1$ (b.c.u), and $\alpha = 0.5$, and $\gamma_{a,b} = \gamma, \forall a \in \mathcal{S} \cup \mathcal{R}, \forall b \in \mathcal{R} \cup \{d\}$, and $a \neq b$. As a benchmark, we consider $(3, 3, 1)$-OMAMRC without feedback [93, 97] in the same configuration, each SDF relay is given one round in the second phase. Fig. 5.2 shows the common outage probability at the destination $\Pr\{\mathtt{E}_{T_{max}}\}$. It can be seen that all the cooperative schemes (with and without feedback) has the same diversity order which equals to $L+1$. Fig. 5.3, shows the throughput η of the different feedback strategies. We notice that:

- The simplest feedback strategy which only requires common ACK/NACK has considerable improvements in terms of throughput over OMAMRC without feedback;
- The throughput of the feedback strategies based on JNCC/JNCD outperform their counterparts that rely on DCC/JDCD. This can be induced directly from equations (5.7) to (5.11);

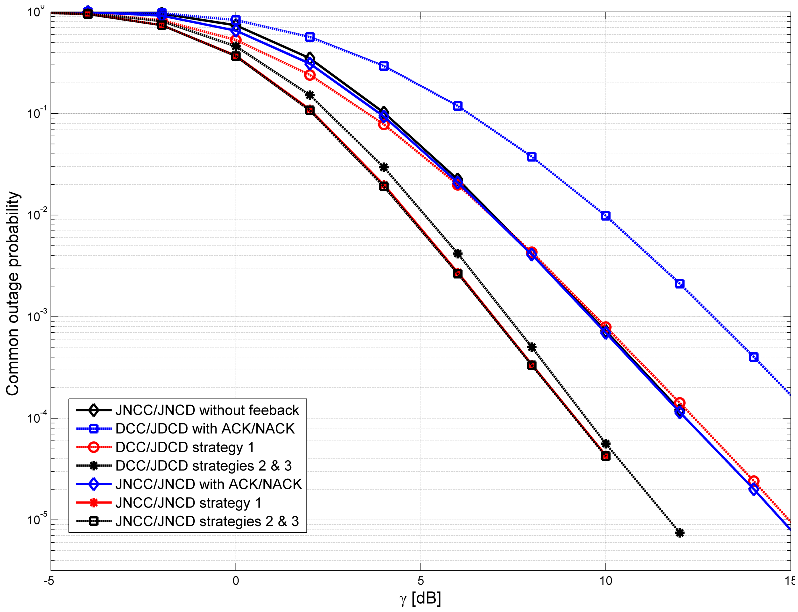


Figure 5.2: The common outage probability $\Pr\{\mathtt{E}_{T_{max}}\}$ of the different feedback strategies in $(3, 3, 1)$-OMAMRC.

- Both strategies 2 and 3 have the same performance in both JNCC/JNCD and DCC/JDCD. This is because with very high probability strategy 2 gives the subset $\hat{\mathcal{S}}_t = \bar{\mathcal{S}}_{d,t-1} \cap \mathcal{S}_{\hat{a}_t,t-1}$ for each selected node as an output of its selection process in (5.13). This remark is an advantage of strategy 3 over strategy 2 since it requires less feedback load.

Figure 5.4, shows the effective throughput of the different feedback strategy in the framework of JNCC/JNCD. Each strategy has its own load factor which is calculated assuming $C = 0.1(b/c.u)$, and $N_2 = 256$. It can be seen that:

- All the feedback strategies still have better performance than OMAMRC without feedback;
- Strategy 3 has better performance than strategy 2 since it has smaller load factor.
- Strategy 1 becomes the best in this scenario. This is because it has mush smaller load factor than strategy 2 and 3, also because the R-R and S-R links are symmetric.

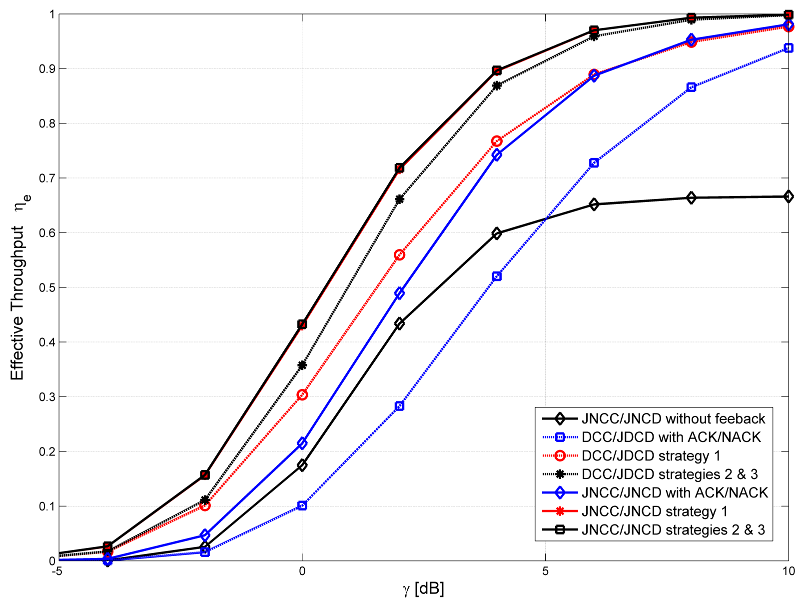


Figure 5.3: The throughput η of the different feedback strategies in $(3, 3, 1)$-OMAMRC, where $T_{max} = 3$, and $\alpha = 0.5$.

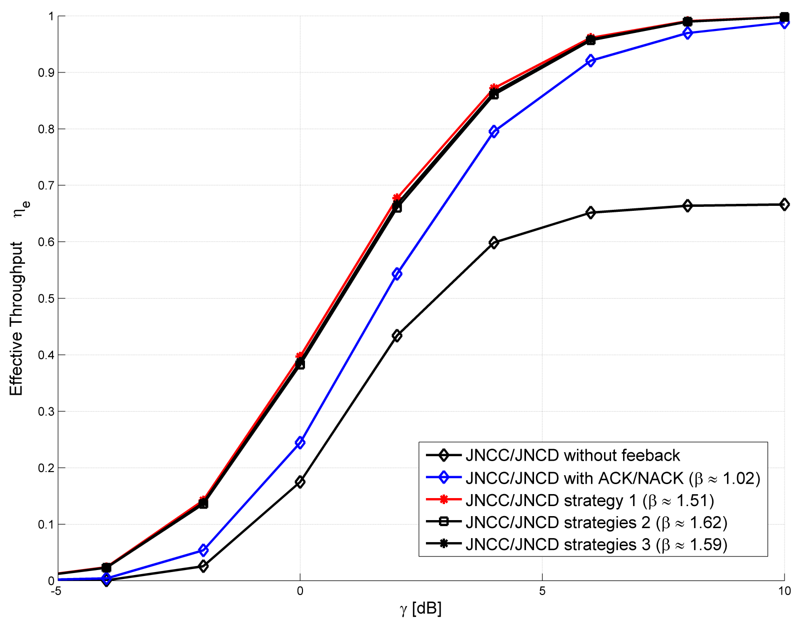


Figure 5.4: The effective throughput η_e of the different JNCC/JNCD feedback strategies in $(3, 3, 1)$-OMAMRC, where $C = 0.1$ (b/c.u), $N_2 = 256$ channel uses , where $T_{max} = 3$, and $\alpha = 0.5$.

CHAPTER 6

Conclusion and research perspectives

6.1 Conclusion

The main contributions that have been achieved in the course of this work are described in the following:

- We have investigated cooperative strategies for the slow-fading half-duplex MAMRC which use static SDF relaying and JNCC/JNCD framework and allow interference among different nodes. We have derived the individual and common outage events. Then, We described in detail when and how JNCC is performed at the relays, in combination with SR, and how JNCD can be efficiently implemented with the sum-product algorithm at the destination. Two different types of linear network coding have been proposed, one operating on the binary field (BI-XOR) and another operating on higher-order Galois fields (GFNC), and their performance compared in a variety of scenarios. BI-XOR based network coding represents an appealing alternative to GFNC, although it does not achieve the full diversity order, due to its inherent flexibility and implementation simplicity. Our link level simulations show that, for the same averaged energy per source, SDF relaying in MAMRC can achieve a considerable gain with respect to direct transmission (without relays) both in terms of reliability (outage/BLER) and transmission rate (achievable rates). But, yet, these gains need to be confirmed through a complete system simulation which should be one of the next steps of this work. In some communication scenarios, which require at least two independent packets to be forwarded to their final destinations through a relay or more, it is straightforward to use the proposed coding schemes of static SDF

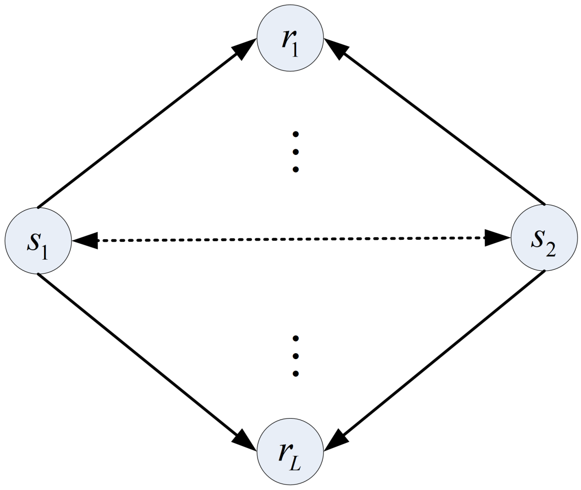


Figure 6.1: The Two-Way Multiple Relay Channel (TWMRC).

relaying. For example:

- Two-Way Multiple Relay Channel (TWMRC):
  In TWMRC, two nodes, denoted by n_1,and n_2, want to send their packets to each other with the help of $L \geq 1$ relays (see fig. 6.1). In general, a direct link between two nodes excites and the relays are used to increase the reliability of the communication. However, if the two nodes are half-duplex and transmit simultaneously then they cannot listen to each other and in this case there is no direct link between n_1 and n_2.
- Broadcast Multiple Relay Channel (BMRC):
  In BMRC, (see fig. 6.2), a source wants to send $M \geq 2$ different packets to M destinations in a round robin fashion (a packet in each time slot is transmitted) with the help of L relays. At the end of the M time slots, the relays are used to increase the reliability of this communication scenario. Each relay could have its own time slot (orthogonal) or all the relay could transmit simultaneously (non-orthogonal).

- We have investigated and proposed a dynamic cooperative strategy, namely dynamic SDF, for the slow-fading half-duplex MAMRC which allows each relay to have a dynamic listing phase and to cooperate with a subset of correctly decoded sources. We have derived the individual and common outage events for certain selection strategies. Then, We described in detail when and how JNCC is performed at the relays and how JNCD can be efficiently implemented with the sum-product algorithm at the destination.

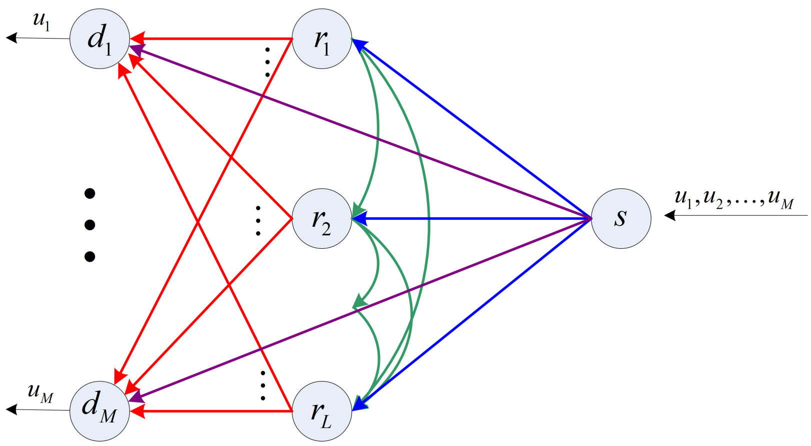


Figure 6.2:

- Finally, we have proposed and compared different types of IR-HARQ cooperative strategies for slow-fading OMAMRC with half-duplex SDF relays. A method to take the effect of the feedback and the in-band signaling on the throughput was proposed. It was shown that even the simplest feedback strategy witch rely on common ACK/NACK can improve the throughput enormously compared to cooperative transmission without feedback and direct transmission (without retransmission rounds).

6.2 Research perspective

Some possible directions for future research are listed below:

- Soft SDF (SSDF) could be an interesting relaying strategy, especially in Orthogonal MAMRC. Unlike SDF, an SSDF relay is not restricted to only help the correctly decoded messages. It can select non correctly decoded messages if they contain an acceptable percentage of errors, which is guarantee using a reliability check function (see [140] for example of such a function). We paved this direction of research in [141] by proposing a static SSDF relay that could be used in a wireless network with at least two independent packets $\mathbf{u}_1$, and $\mathbf{u}_2$ to be transmitted through the SSDF relay to their final destinations. The proposed static SSDF relay works as follow:

  1. Estimate the packets from the received signals using soft decoding techniques.
  2. Find the perfect packets (the ones with no errors) using the CRC checks.

3. Find the good packets (the packets that are not perfect, but have an acceptable reliability).
4. Select a set of packets for cooperation
   - If all the selected packets are perfect (have passed the CRC check) then it work as an SDF relay (as described in chapter 3).
   - If some of the selected packets are good (have passed the reliability threshold check but not the CRC check) then it compressed all LLR vectors of the packets using techniques similar to one presented in [84] (the LLR of the perfect packets messages are set to +/- infinity).
   - If no packet is selected (no packet is perfect or good) then the relay keeps silent.

The reliability check is a mechanism to control the bit error rate that a relay forward to the destination hence reduce harmful interference in the wireless network (The CRC check is an extreme case where no errors are allowed). A fair and objective compassion between SSDF and other relaying strategies needs further investigation. Also, method to optimize the reliability thresholds for each relay is needed, which might require the combination of SSDF with feedback strategies similar to the one presented in chapter 5.

- Full-duplex relaying could be a possible direction to investigate. In this direction of research, we propose two patents that describe a new full duplex relaying protocol namely Full-Duplex Dynamic Selective Decode-and-Forward (FD-DSDF) for NO-MAMRC. The advantages of the proposed FD-DSDF are: (1) It prevents error propagation from the relay to the destination; (2) It reduces the energy consumption at the relay and limits the interference within the network (the relay is always helpful when it cooperates); (3) It is designed for non-orthogonal channel access so it guarantee maximum throughput. (this is not an advantage in orthogonal MAMRC) (4) It completely exploits the full-duplex property. during a given time slot, as soon as the relay correctly decodes a message of a source, it immediately transmits a representative signal that help this message and meanwhile it keeps trying to decode the other messages of the other sources. (6) The design of the relay decoding and destination encoding are flexible and can be modified to handle the complexity constraints imposed by the system designer.

  To describe the general principle and for simplicity we consider (2,1,1)-NOMAMRC, two sources S_1, S_2 and one relay R_1. The sources transmit their messages during T time slots. Each time slot is divided into B sub-slots, the transmitted signal during

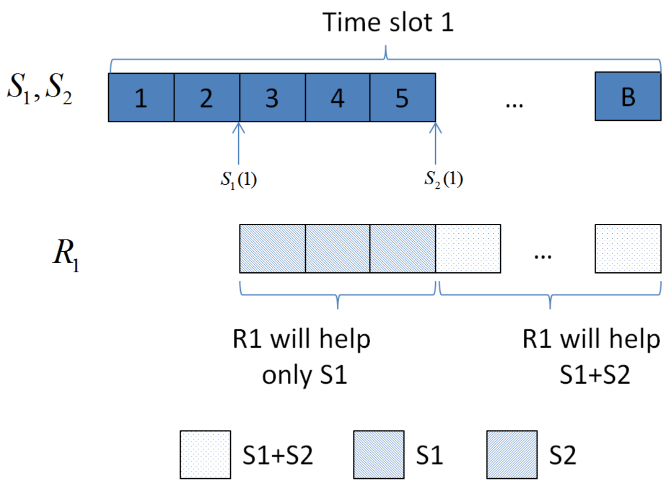


Figure 6.3: A possible cooperation scenario in FD-DSDF for NO-MARC, when $T = 1$

each sub-slot is designed to add extra redundancy to the previous sub-slots of the same time slot, hence, a receiver which receives more of these sub-slots will have a better chance to correctly decode the messages of the sources.

1. We describe the relaying function within the first time slot $t = 1$.
   At the end of each sub-slot b, where $b \in \{1, \ldots, B\}$, the relay tries to decode the messages of the sources (if they are not correctly decoded yet). If the relay correctly decodes the sources it will send a useful signal that helps both sources, if it correctly decodes one source it will send a useful signal to help this source, and if it does not decode any source it will remain silent.
   Example (see fig. 6.3): Let us consider that the relay is able to decode S_1 then S_2 at the end of sub-slot 2, and 5, respectively. Since the relay is full-duplex it can help S_1 during sub-slots $3, 4, 5$, and continue to listen meanwhile. At the end of sub-slot 5, the relay does two operations: (1) it stops listening because it has correctly decoded all the messages of the sources; (2) it starts to help both sources by sending a network coded message generated from the messages of S_1 and S_2 .

2. During time slot $t > 1$.
   At the end of each sub-slot b, where $b \in \{1, \ldots, B\}$, the relay tries to decode the messages of the sources (if they are not correctly decoded yet). If the relay correctly decodes the sources it will send a useful signal to help both sources, if it correctly decodes one source it will send a useful signal to help this source, and if it does not decode any source it will continue to help the correctly decoded

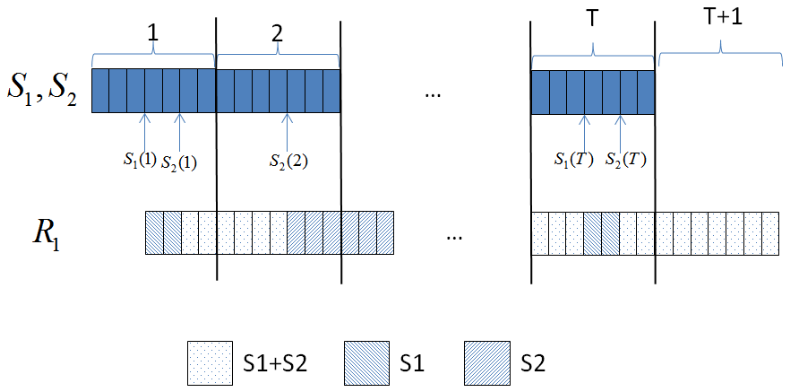


Figure 6.4: A possible cooperation scenario in FD-DSDF for NO-MARC

messages sent during previous time slots, if any.
Example (see fig. 6.4): Let us consider that during the first time slot $t = 1$, the relay was able to decode S_1 then S_2 at the end of sub-slot 3, and 5, respectively. And during time slot $t = 2$, the relay is able to correctly decode the message of S_2 at the end of sub-slot 4. In this case, during the sub-slots $1, 2, 3, 4$ in time slot $t = 2$, the relay will continue to help the messages of S_1 and S_2 of the first time slot. From sub-slot 5 in time slot $t = 2$ the relay will start to help the message of S_2 of the second time slot until new message is correctly decoded, and so on.

The detailed description with practical code implementation exist in [142, 143].

- Combining IR-HARQ with other relaying strategies such as D-SDF and SSDF represent an interesting research direction.

- Non-binary channel coding at the sources and relays could be investigated. In this case, the coding and modulation schemes at the sources and relays are defined in the non-binary field where the network coding coefficients are chosen. This may improve the performance, but at the same time it could be restrictive in terms of spectral efficiency. A possible design based on LDPC code was treated in [89] for OMAMRC. Further code design could be conducted based on non binary turbo coding.

APPENDIX A

Proof of proposition 4

Using the assumption that the fading coefficients $\mathbf{h} = \{\mathbf{h}_{a,b}\}$, where $a \in \{s, r\}$, and $b \in \{r, d\}$ are mutually independent. This assumption is important to insure that the sets $\mathcal{S}_r$ are mutually independent, we can write eq. (3.13) as

$$
\begin{aligned}
P_{out,d}^{com} &= \sum_{\mathcal{S}_{r_1} \subseteq \mathcal{S}} \cdots \sum_{\mathcal{S}_{r_L} \subseteq \mathcal{S}} \Pr\{\mathcal{S}_{r_1}\} \ldots \Pr\{\mathcal{S}_{r_L}\} \Pr\{\mathcal{E}_d | \mathcal{S}_{r_1}, \ldots, \mathcal{S}_{r_L}\} && \text{(A.1a)} \\
&\overset{(a)}{\leq} \sum_{\mathcal{S}_{r_1} \subseteq \mathcal{S}} \cdots \sum_{\mathcal{S}_{r_L} \subseteq \mathcal{S}} \Pr\{\mathcal{S}_{r_1}\} \ldots \Pr\{\mathcal{S}_{r_L}\} \left(\sum_{\mathcal{U} \subseteq \mathcal{S}} \Pr\{\mathtt{F}_{d,\mathcal{S}}(\mathcal{U}) | \mathcal{S}_{r_1}, \ldots, \mathcal{S}_{r_L}\} \right) && \text{(A.1b)} \\
&= \sum_{\mathcal{U} \subseteq \mathcal{S}} \sum_{\mathcal{R}_u \subseteq \mathcal{R}} \Pr\{\mathcal{R}_u\} \Pr\{\mathtt{F}_{d,\mathcal{S}}(\mathcal{U}) | \mathcal{R}_u, \mathcal{S}_{r_1}, \ldots, \mathcal{S}_{r_L}\}, && \text{(A.1c)}
\end{aligned}
$$

where $\mathcal{R}_u \triangleq \{r \in \mathcal{R} : \mathcal{S}_r \cap \mathcal{U} \neq \emptyset\}$ is the set of relays whose signals are to be jointly decoded with the sources belonging to $\mathcal{U}$, $\Pr\{\mathcal{R}_u\}$ is the probability that the set of relays whose signals are to be jointly decoded with the sources belonging to $\mathcal{U}$ equals $\mathcal{R}_u$, and $\Pr\{\mathcal{S}_r\}$ is the probability that the set of sources that relay r cooperate with equals $\mathcal{S}_r$. (a) Follows from the union bound. Using (3.10) and (3.14) we can write

$$
P_1 \triangleq \Pr\{\mathtt{F}_{d,\mathcal{S}}(\{\mathcal{U}\}) | \mathcal{R}_u, \mathcal{S}_{r_1}, \ldots, \mathcal{S}_{r_L}\} = \Pr\left(\prod_{i=1}^{T} \Big(1 + \sum_{a \in \mathcal{T}_i \cap (\{\mathcal{U}\} \cup \mathcal{R}_u)} Z_{a,d}\Big)^{\alpha_i} \leq 2^R \right), \quad \text{(A.2)}
$$

where $Z_{a,d} = \gamma_{a,d}||\mathbf{h}_{a,d}||^2 \sim Gamma(2m_d, \gamma_{a,d})$, where $a \in \{s, r\}$. Using the assumption that $\alpha_i > 0$, and $\cup_i \mathcal{T}_i = \mathcal{S} \cup \mathcal{R}$ (if a node is not given the possibility to transmit at lest in one transmission interval then it is not in the MAMRC) we can write

$$P_1 = \int \cdots \int_{\mathcal{A}} \prod_{s \in \mathcal{U}} p(z_{s,d}) d(z_{s,d}) \prod_{r \in \mathcal{R}_u} p(z_{r,d}) d(z_{r,d}) \tag{A.3}$$

where $\mathcal{A} \triangleq \{(z_{s,d} \geq 0, z_{r,d} \geq 0 : s \in \mathcal{U}, r \in \mathcal{R}_u) : \prod_{i=1}^{T} \left(1 + \sum_{a \in \mathcal{T}_i \cap (\{\mathcal{U}\} \cup \mathcal{R}_u)} z_{a,d}\right)^{\alpha_i} \leq 2^R\}$, $p(z_{a,d})$ is the p.d.f of the random variable $Z_{a,d}$.

$$P_1 = \int \cdots \int_{\mathcal{A}} \prod_{s \in \mathcal{U}} \frac{c_1}{\gamma_{s,d}^{m_d}} z_{s,d}^{m_d - 1} e^{-\frac{z_{s,d}}{2\gamma_{s,d}}} d(z_{s,d}) \prod_{r \in \mathcal{R}_u} \frac{c_1}{\gamma_{r,d}^{m_d}} z_{r,d}^{m_d - 1} e^{-\frac{z_{r,d}}{2\gamma_{r,d}}} d(z_{r,d}), \tag{A.4}$$

where $c_1 = 1/(2^{m_d} m_d!)$. We consider that $\gamma_{a,b} = c_{a1,b}\gamma$ where $c_{a1,b} \in \mathbb{R}^+$ is some real positive constant, with respect to γ. Then,

$$P_1 = \frac{c_2}{\gamma^{(|\mathcal{U}| + |\mathcal{R}_u|)m_d}} \int \cdots \int_{\mathcal{A}} \prod_{s \in \mathcal{U}} z_{s,d}^{m_d - 1} e^{-\frac{z_{s,d}}{2\gamma_{s,d}}} d(z_{s,d}) \prod_{r \in \mathcal{R}_u} z_{r,d}^{m_d - 1} e^{-\frac{z_{r,d}}{2\gamma_{r,d}}} d(z_{r,d}) \tag{A.5a}$$

$$\overset{(a)}{\leq} \frac{c_2}{\gamma^{(|\mathcal{U}| + |\mathcal{R}_u|)m_d}} \int \cdots \int_{\mathcal{A}} \prod_{s \in \mathcal{U}} z_{s,d}^{m_d - 1} d(z_{s,d}) \prod_{r \in \mathcal{R}_u} z_{r,d}^{m_d - 1} d(z_{r,d}), \tag{A.5b}$$

$$\overset{(b)}{=} \frac{c_3}{\gamma^{(|\mathcal{U}| + |\mathcal{R}_u|)m_d}}, \tag{A.5c}$$

where c_2, c_3 are some constants in $\mathbb{R}^+$. (a) Follows from the fact that $exp(-x/\gamma) \leq 1$ for all $x \geq 0$ and $\gamma \geq 0$. (b) Follows because the set $\mathcal{A}$ is a compact subset of a hyper-cube $\mathcal{H} \triangleq \{(z_{a,d} : a \in \mathcal{U} \cup \mathcal{R}_u) : \ 0 \leq z_{a,d} \ \leq 2^{R/\alpha_{a,d}} - 1\}$, where $\alpha_{a,d} = \sum_{i=1}^{T} \alpha_i \mathbb{1}_{\{a \in \mathcal{T}_i\}}$ and the function inside the integral is bounded on $\mathcal{H}$.

The probability that $\mathcal{R}_u$ is the set of relays whose signals are to be jointly decoded with the sources $\mathcal{U}$ is given by

$$\Pr\{\mathcal{R}_u\} \overset{(a)}{=} \prod_{r \notin \mathcal{R}_u} \left(\prod_{s \in \mathcal{U}} p_{out}^{s,r}\right) \prod_{r \in \mathcal{R}_u} \left(1 - \left(\prod_{s \in \mathcal{U}} p_{out}^{s,r}\right)\right) \doteq \frac{c_5}{\gamma^{m_r |\mathcal{U}|(L - |\mathcal{R}_u|)}}, \tag{A.6}$$

where $p_{out}^{s,r}$ is the outage probability of source s at relay r, and $p_{out}^{s,r} \doteq \gamma^{-m_r}$ (see [144]). (a) Follows from the fact that a relay r belong to $\mathcal{R}_u$ iff at least one source in the set $\mathcal{U}$ is not in outage at r (alternatively speaking if r is cooperating with at least one source from $\mathcal{U}$). Using (A.1),(A.5), and (A.6), we can write

$$p_{out}^{com} \dot{\leq} \sum_{\mathcal{U}\subseteq\mathcal{S}} \sum_{\mathcal{R}_u\subseteq\mathcal{R}} \frac{c_5}{\gamma^{m_r|\mathcal{U}|(L-|\mathcal{R}_u|)}} \frac{c_3}{\gamma^{(|\mathcal{U}|+|\mathcal{R}_u|)m_d}}. \tag{A.7}$$

Hence, the slowest decay rate of (A.7) is given by

$$D_d^{min} = \min_{i=1,\ldots,M} \min_{j=0,1,\ldots,L} (i(m_r L + m_d) + j(m_d - i m_r)) \tag{A.8}$$

The result of previous minimization is given by

- $m_d > m_r$, then $D_d^{min} = m_r L + m_d$.
- $m_d \leq m_r$ then $D_d^{min} = m_d(L+1)$.

Combining with the fact that $\Pr\{\mathcal{E}_d|\mathcal{S}_{r_1},\ldots,\mathcal{S}_{r_L}\} \geq \Pr\{\mathtt{F}_{d,\mathcal{S}}(\mathcal{U}^*)|\mathcal{R}_u,\mathcal{S}_{r_1},\ldots,\mathcal{S}_{r_L}\}$, where $\mathcal{U}^*$ is the set of sources that achieve (A.8) (any set with cardinality 1), we finish the proof of proposition 4.

APPENDIX B

Proof of Proposition 5

We aim at deriving the conditions on the rates which allow the destination to decode the packets of the sources in $\mathcal{I}^c = \mathcal{S} \backslash \mathcal{I}$ knowing that the signals of $\mathcal{I}$ are interference. First, we consider that no relay was able to cooperate during the period, i.e., $\Delta_{r_1} \geq \Delta$ and $A = 0$. In this case, at d, we have an $|\mathcal{I}^c|$-sender MAC interfered by $\mathcal{I}$ signals, and the rates must satisfy

$$R_{\mathcal{U}} \leq I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}) \quad \text{for all } \; \mathcal{U} \subseteq \mathcal{I}^c, \tag{B.1}$$

where $\mathcal{U}_c = \mathcal{I}^c \setminus \mathcal{U}$, $R_{\mathcal{U}} = |\mathcal{U}|R$. Then, we consider that only the first relay r_1 was able to cooperate within the available transmission period, i.e., $\Delta_{r_1} < \Delta \leq \Delta_{r_2}$ $(A = 1)$. In this case, at destination d, we have a first phase $|\mathcal{I}^c|$-sender MAC interfered by $\mathcal{I}$ signals, and a second phase $|\mathcal{I}^c|$-sender MAC interfered by $\mathcal{I}$ and $\mathcal{R}_1^I$ signals (the signals of the relays $\mathcal{R}_1^U$ are codewords for the packets of the sources in $\mathcal{I}^c$ and are treated jointly with them), and the rates must satisfy

$$R_{\mathcal{U}} \leq \alpha_1^d I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}) + \bar{\alpha}_1^d I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}_1^U}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}_1^K}) \quad \text{for all } \; \mathcal{U} \subseteq \mathcal{I}^c, \tag{B.2}$$

where $\alpha_1^d = \Delta_{r_1}/\Delta$ is the time sharing factor, and $\bar{\alpha}_1^d = 1 - \alpha_1^d$. Following the same reasoning, we find that, for an arbitrary value of $A \leq L$, the rates must satisfy

$$R_{\mathcal{U}} \leq \sum_{j=0}^{L} (\alpha_{j+1}^d - \alpha_j^d) I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}_j^U}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}_j^K}) \quad \text{for all } \; \mathcal{U} \subseteq \mathcal{I}^c, \tag{B.3}$$

where $\alpha_0^d = 0$, $\alpha_i^d = \min\{\frac{\Delta_{r_i}}{\Delta}, 1\}$, $\forall i = 1, \ldots, L$, and $\alpha_{L+1}^d = 1$ are the time sharing factors. As a consequence, the maximum possible rate which allows the destination d to decode all the packets of the sources in $\mathcal{I}^c$ (common) knowing that the signals in $\mathcal{I} \subset \mathcal{S}$ are interference is given by

$$R_{\mathcal{I}^c}^d = \min_{\mathcal{U} \subseteq \mathcal{I}^c} \frac{\sum_{j=0}^{L} (\alpha_{j+1}^d - \alpha_j^d) I(\mathbf{x}_{\mathcal{U}}, \mathbf{x}_{\mathcal{R}_j^U}; \mathbf{y}_d | \mathbf{x}_{\mathcal{U}_c}, \mathbf{x}_{\mathcal{R}_j^K})}{|\mathcal{U}|}, \tag{B.4}$$

■

APPENDIX C

Proof of Proposition 6

The minimum listening period of the first cooperating relay r_1 is given by

$$\Delta_{r_1} = \min_{r \in \mathcal{R}} \frac{K}{R^r_{\mathcal{S}}}, \tag{C.1}$$

where $R^r_{\mathcal{S}}$ is given by (4.13). The minimum listening period of the second cooperating relay r_2 is given by

$$\Delta_{r_2} = \min_{r \in \mathcal{R}^c_1} \frac{K}{R^r_{\mathcal{S},\mathcal{R}_1}}, \tag{C.2}$$

where $R^r_{\mathcal{S},\mathcal{R}_1}$ is the maximum symmetric rate which allows r to decode all the packets of the sources with the help of the relay r_1 and $\mathcal{R}^c_1 = \mathcal{R} \backslash \mathcal{R}_1$. Assuming that JNCC was employed, the relay r will jointly decode the received signals of the sources and r_1. Hence, we can write

$$\begin{aligned} R^r_{\mathcal{S},\mathcal{R}_1} &\overset{(a)}{=} \min_{\mathcal{U} \subseteq \mathcal{S}} \frac{\alpha^r_{r_1} I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_r | \mathbf{x}_{\mathcal{U}^c}) + \bar{\alpha}^r_{r_1} I(\mathbf{x}_{\mathcal{U}}, x_{r_1}; \mathbf{y}_r | \mathbf{x}_{\mathcal{U}^c})}{|\mathcal{U}|} && \text{(C.3a)} \\ &\overset{(b)}{=} \min_{\mathcal{U} \subseteq \mathcal{S}} \frac{I(\mathbf{x}_{\mathcal{U}}; \mathbf{y}_r | \mathbf{x}_{\mathcal{U}^c}) + \bar{\alpha}^r_{r_1} I(x_{r_1}; \mathbf{y}_r | \mathbf{x}_{\mathcal{S}})}{|\mathcal{U}|} && \text{(C.3b)} \end{aligned}$$

where $\alpha^r_{r_1} = \frac{\Delta_{r_1}}{\Delta_r} \overset{(c)}{\in} \left[\frac{R^r_{\mathcal{S}}}{R^{r_1}_{\mathcal{S}}}, 1\right]$ and $\bar{\alpha}^r_{r_1} = 1 - \alpha^r_{r_1}$. (a) follows from the fact that during Δ_{r_1} channel uses the relay r will only receive the signals of the sources, while during $\Delta_r - \Delta_{r_1}$ channel uses, the relay r will receive, in addition to the signals of the sources, a helpful signal from r_1. (b) follows from the chain rule of mutual information. (c) follows from the following two extreme cases

- r has the same decoding capability as r_1, which means that $\Delta_r = \Delta_{r_1}$, $R_{\mathcal{S}}^{r_1} = R_{\mathcal{S}}^{r}$, and $\alpha_{r_1}^{r} = 1$;

- r cannot make use of the signal of r_1, i.e., there is no link between r and r_1. In this case, we have $I(x_{r_1}; \mathbf{y}_r | \mathbf{x}_{\mathcal{S}}) = 0$, $\Delta_r = K/R_{\mathcal{S}}^{r}$, and $\alpha_{r_1}^{r} = R_{\mathcal{S}}^{r_1}/R_{\mathcal{S}}^{r}$.

It is not obvious to obtain the expression of $\Delta_r = K/R_{\mathcal{S},\mathcal{R}_1}^{r}$ analytically, since $\bar{\alpha}_{r_1}^{r}$ is a function of Δ_r and as such cannot be taken outside the min in (C.3b). Instead, the value evaluated numerically, by scanning the range $[K/R_{\mathcal{S}}^{r}, K/R_{\mathcal{S}}^{r_1}]$. We iterate the reasoning and find that the minimum listening period of the i^{th} cooperating relay r_i is

$$\Delta_{r_i} = \min_{r \in \mathcal{R}_{i-1}^{c}} \frac{K}{R_{\mathcal{S},\mathcal{R}_{i-1}}^{r}}, \tag{C.4}$$

■

APPENDIX D

Mutual information calculation for different types of input distribution

Let's consider the $|\mathcal{S}|$-users block fading MAC described before where the received signal at the destination is written as

$$\mathbf{y}_d = \sum_{s\in\mathcal{S}} \mathbf{h}_{sd} x_s + \mathbf{n} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{D.1}$$

where $\mathbf{H} = \mathbf{h}_{\mathcal{S}d}$, and $\mathbf{x} = [x_{\mathcal{S}}]^T$, We removed the time index for simplicity.

Let's define the independent input random variables $x_s \sim p(x_s)$, and the associated independent output random vector $\mathbf{y}_d$, whose channel transition conditional pdf is $p(\mathbf{y}_d|x_{\mathcal{S}}, \mathbf{H}) = \mathcal{CN}(\mathbf{H}\mathbf{x}, N_0\mathbf{I})$. In this appendix, we want to derive the expression of instantaneous mutual information $I(x_{\mathcal{U}}; \mathbf{y}_d|x_{\mathcal{U}_c}, \mathbf{H})$ for two different types of input distribution a) Gaussian i.i.d and b) discrete i.i.d inputs, where $\mathcal{U}_c = (\mathcal{S}\backslash\mathcal{I})\backslash\mathcal{U}$, $\mathcal{I}$ is the set of interfering users, and $\mathcal{U}$ is the set of known users (for example decoded by Successive Interference Decoder SIC). the receive signal at the destination could be written as

$$\mathbf{y}_d = \sum_{s\in\mathcal{U}} \mathbf{h}_{sd} x_s + \sum_{s\in\mathcal{U}^c} \mathbf{h}_{sd} x_s + \sum_{s\in\mathcal{I}} \mathbf{h}_{sd} x_s + \mathbf{n} \tag{D.2}$$

Where the signals in $\mathcal{I}$ are interference and will be treated as noise. the signals in $\mathcal{U}^c = \mathcal{I}^c\backslash\mathcal{U}$ are perfectly known. equation (D.2) could be written as

$$\tilde{\mathbf{y}}_d = \mathbf{y}_d - \sum_{s\in\mathcal{U}^c} \mathbf{h}_{sd} x_s = \mathbf{H}_u \mathbf{x}_u + \mathbf{w} \tag{D.3}$$

where $\mathbf{H}_u = \mathbf{h}_{\mathcal{U}d}$, $\mathbf{x}_u = [x_{\mathcal{U}}]^T$, and $\mathbf{w}$ is the noise plus interference and given by

$$\mathbf{w} = \mathbf{H}_I \mathbf{x}_I + \mathbf{n} \tag{D.4}$$

where $\mathbf{H}_I = \mathbf{h}_{\mathcal{I}d}$, $\mathbf{x}_I = [x_{\mathcal{I}}]^T$.

D.1 Gaussian i.i.d inputs

In this case, the vectors $\mathbf{x}_u$ and $\mathbf{x}_I$ are circular symmetric Gaussian random vectors, then using [145, lemma 1, 2, and 3] we can write.

$$\begin{aligned} I(x_{\mathcal{U}}; \mathbf{y}_d | x_{\mathcal{U}^c}, \mathbf{H}) &= I(x_{\mathcal{U}}; \tilde{\mathbf{y}}_d | \mathbf{H}) \\ &= h(\tilde{\mathbf{y}}_d | \mathbf{H}) - h(\tilde{\mathbf{y}}_d | x_{\mathcal{U}}, \mathbf{H}) \\ &= \log \left(\frac{\det\left(N_0 \mathbf{I}_{N_D} + \mathbf{H}_I \mathbf{H}_I^H + \mathbf{H}_u \mathbf{H}_u^H\right)}{\det\left(N_0 \mathbf{I}_{N_D} + \mathbf{H}_I \mathbf{H}_I^H\right)} \right) \end{aligned} \tag{D.5}$$

D.2 Discrete i.i.d. inputs

In this case, discrete channel inputs x_i are chosen from the constellations $\mathcal{X}_i$ of order 2^{q_i}. We assume uniform input distributions. Thus, $p(x_i) = 2^{-q_i}$. The instantaneous mutual information is derived numerically as

$$I(x_{\mathcal{U}}; \mathbf{y}_d | x_{\mathcal{U}^c}, \mathbf{H}) = I(x_{\mathcal{U}}; \tilde{\mathbf{y}}_d | \mathbf{H}) \overset{(a)}{=} H(x_{\mathcal{U}}) - H(x_{\mathcal{U}} | \tilde{\mathbf{y}}_d, \mathbf{H}) \tag{D.6}$$

(a) Follows from the independence of users' inputs and channel state. The first term is calculated as

$$H(x_{\mathcal{U}}) = -\mathbb{E}\left[\log_2 p(x_{\mathcal{U}})\right] \overset{(a)}{=} -\mathbb{E}\left(\log_2 \prod_{s \in \mathcal{U}} p(x_i)\right) = -\mathbb{E}\left(\log_2 \prod_{s \in \mathcal{U}} 2^{-q_i}\right) = \log_2 M_{\mathcal{U}}$$

Where $M_{\mathcal{U}} = \prod_{s\in\mathcal{U}} 2^{q_i}$, (a) follows from the fact that the users inputs are mutually independent. The second term is calculated as

$$
\begin{aligned}
H(x_{\mathcal{U}}|\tilde{\mathbf{y}}_d,\mathbf{H}) &= \frac{1}{M_{\mathcal{U}}}\sum_{x_{\mathcal{U}}}\mathbb{E}\left(\log_2\frac{1}{p(x_{\mathcal{U}}|\tilde{\mathbf{y}}_d,\mathbf{H})}\right)\\
&= \frac{1}{M_{\mathcal{U}}}\sum_{x_{\mathcal{U}}}\mathbb{E}\left(\log_2\frac{p(\tilde{\mathbf{y}}_d|\mathbf{H})}{p(\mathbf{y}_d|x_{\mathcal{U}},\mathbf{H})p(x_{\mathcal{U}})}\right)\\
&= \frac{1}{M_{\mathcal{U}}}\sum_{x_{\mathcal{U}}}\mathbb{E}\left(\log_2\frac{\sum\limits_{\tilde{x}_{\mathcal{U}\cup\mathcal{I}}} p(\tilde{\mathbf{y}}_d|\tilde{x}_{\mathcal{U}\cup\mathcal{I}},\mathbf{H})p(\tilde{x}_{\mathcal{U}\cup\mathcal{I}})}{\sum\limits_{\hat{x}_{\mathcal{I}}} p(\mathbf{y}_d|\hat{x}_{\mathcal{I}},x_{\mathcal{U}},\mathbf{H})p(\hat{x}_{\mathcal{I}})p(x_{\mathcal{U}})}\right)\\
&= \frac{1}{M_{\mathcal{U}}}\sum_{x_{\mathcal{U}}}\mathbb{E}\left(\log_2\frac{\sum\limits_{\tilde{x}_{\mathcal{U}\cup\mathcal{I}}} p(\tilde{\mathbf{y}}_d|\tilde{x}_{\mathcal{U}\cup\mathcal{I}},\mathbf{H})}{\sum\limits_{\hat{x}_{\mathcal{I}}} p(\mathbf{y}_d|\hat{x}_{\mathcal{I}},x_{\mathcal{U}},\mathbf{H})}\right)
\end{aligned}
$$

The last expectation is with respect to $p(\tilde{\mathbf{y}}_d|x_{\mathcal{U}},\mathbf{H})$.

$$
\begin{aligned}
H(x_{\mathcal{U}}|\tilde{\mathbf{y}}_d,\mathbf{H}) &= \frac{1}{M_{\mathcal{U}}}\sum_{x_{\mathcal{U}}}\frac{1}{M_{\mathcal{I}}}\sum_{x_{\mathcal{I}}}\mathbb{E}_{\mathbf{n}}\left(\log_2\frac{\sum\limits_{\tilde{x}_{\mathcal{U}\cup\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_u\mathbf{x}_{\mathcal{U}}+\mathbf{h}_I x_{\mathcal{I}}-\mathbf{h}_I\tilde{x}_{\mathcal{I}}-\mathbf{h}_u\tilde{\mathbf{x}}_{\mathcal{U}}\|^2}}{\sum\limits_{\hat{x}_{\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_I x_{\mathcal{I}}-\mathbf{h}_I\hat{x}_{\mathcal{I}}\|^2}}\right)\\
&= \frac{1}{M_{\mathcal{U}\cup\mathcal{I}}}\sum_{x_{\mathcal{U}\cup\mathcal{I}}}\mathbb{E}_{\mathbf{n}}\left(\log_2\frac{\sum\limits_{\tilde{x}_{\mathcal{U}\cup\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_u(\mathbf{x}_{\mathcal{U}}-\tilde{\mathbf{x}}_{\mathcal{U}})+\mathbf{h}_I(x_{\mathcal{I}}-\tilde{x}_{\mathcal{I}})\|^2}}{\sum\limits_{\hat{x}_{\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_I(x_{\mathcal{I}}-\hat{x}_{\mathcal{I}})\|^2}}\right)
\end{aligned}
$$

Finally,

$$
I(x_{\mathcal{U}};\mathbf{y}_d|x_{\mathcal{U}^c}) = \log_2 M_{\mathcal{U}} - \frac{1}{M_{\mathcal{U}\cup\mathcal{I}}}\sum_{x_{\mathcal{U}\cup\mathcal{I}}}\mathbb{E}_{\mathbf{n}}\left(\log_2\frac{\sum\limits_{\tilde{x}_{\mathcal{U}\cup\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_u(\mathbf{x}_{\mathcal{U}}-\tilde{\mathbf{x}}_{\mathcal{U}})+\mathbf{h}_I(x_{\mathcal{I}}-\tilde{x}_{\mathcal{I}})\|^2}}{\sum\limits_{\hat{x}_{\mathcal{I}}} e^{-\frac{1}{N_0}\|\mathbf{n}+\mathbf{h}_I(x_{\mathcal{I}}-\hat{x}_{\mathcal{I}})\|^2}}\right) \tag{D.7}
$$

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